Private versus public monopoly

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Abstract

We compare private and public monopoly with respect to how much resources they spend on finding out what product varieties people want. We propose a simple model in which a monopolist supplies one variety of a good. This variety is chosen by the monopolist and consumers differ in their valuations of the good and preferences over product varieties. The monopolist does not know the preference distribution, but can, at a cost, acquire more or less precise information about this distribution. We analyze the monopolist's endogenous information acquisition and choice of product variety in the following three scenarios: an unregulated profit-maximizing monopolist, the first-best welfare solution, and a monopolist who maximizes a convex combination of profit and welfare under a budget constraint. Our main finding is that, broadly speaking, public monopoly is preferable in societies with a wide spread in income and/or wealth while private monopoly is better in societies with less inequity.

Keywords: Uncertainty, information acquisition, monopoly, Boiteux-Ramsey price, regulation, inequity.
JEL Classification: D21, D42, D60, L12, L51

1 Introduction

We here address the following question: Does a monopolist make too little, just about right, or too much - from a welfare point of view - to find out what product varieties consumers want? We posit a simple model in which a monopolist chooses a product

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variety, from a continuum range, to supply to the market. Consumers have idiosyncratic Euclidean preferences over product varieties, and differ in their willingness to pay. For example, one rich individual may prefer a certain variety x and is willing to pay a lot for this personal ideal variety, while another, poor, individual may prefer a variety y (which may or may not be the same as x), but is willing to pay less for her personal variety. The monopolist does not know the preference distribution in the population. It only has a vague prior belief about it. However, the monopolist may, at a cost, choose to obtain more precise information, where higher precision costs more. We represent the monopolist's information about demand as a noisy signal about some parameter in the preference distribution. The monopolist may be an unregulated profit maximizer, an unconstrained welfare maximizer ("first-best"), or, more generally, it may have a goal function that is an arbitrary convex combination of profit and welfare.

This paper belongs to the small literature dealing with firms' choice of price and location, or product variety, under uncertainty (see Vives, 1984; Harter, 1997; Casado-Izaga, 2000; Meagher and Zauner, 2004, 2005, 2011; Król, 2012). While all of those papers deal with duopoly under exogenous uncertainty, we here analyze monopoly under endogenous uncertainty.[1] The paper most closely related to us is Meagher (1996). However, that paper does not address the question of optimal information acquisition. Instead, it considers a dynamic environment in which consumer preferences change over time and are not directly observable, and where the firm can conduct market research.

We proceed to first analyze an unregulated profit-maximizing monopolist's information gathering, price setting and choice of product variety (or location). Secondly, we perform the same analysis on an unconstrained welfare-maximizing monopolist, where welfare is defined as the sum of profit and consumer surplus. Such a monopolist will finance its activity by a lump-sum tax on all citizens. Third, and last, we analyze a monopolist who maximizes expected welfare under the constraint that its expected profit reaches an arbitrary pre-specified level. This setting is in line with the classical Ramsey-Boiteux approach (see Ramsey, 1927; Boiteux, 1971), and we obtain results for a wide range of second-best cases.

In our simple model, the monopolist's choice of product variety, at any given level of its uncertainty about consumer preferences, is always socially efficient; irrespective if its goal is profit or welfare or some convex combination thereof, the monopolist always strives to supply a variety that will attract as many consumers as possible, at

[1] In this paper we assume that consumers know their own valuations of the product. If consumers do not know their valuations, the monopolist's pricing may convey information to the consumers and there may be multiple equilibria. Furthermore, if consumers can acquire costly information, then the price in equilibrium might not be perfectly informative. See Bester and Ritzberger (2001) and the references therein. If there are more than one firm in the market, and the consumes do not know the quality or the valuation of the product, consumers' information acquisition choice may endogenize the trading rules, see Bester (1993) for an inference.

any given price. By contrast, the different types of monopoly differ in the extent of their information acquisition. An unregulated profit-maximizing monopolist will typically either spend too little or too much resources on information acquisition, compared with first-best. The wider is the dispersion of consumers' valuations of their ideal product varieties, the further is the unregulated private monopoly from first-best information acquisition. In a society with a wide spread in valuations, the unregulated private monopolist therefore underinvests in information acquisition about consumer preferences, while in societies with little spread in valuations it overinvests, compared to first-best. It is only at an intermediate knife-edge case of dispersion of valuations that an unregulated private monopolist achieves the first best. Broadly speaking, public or regulated private monopoly is preferable in societies with a wide dispersal of valuations, as would be expected in societies with a wide spread in income and/or wealth (say, India). By contrast, in societies with modest spread in valuations, such as would be expected in societies with fairly equal disposable incomes or wealth (say, Sweden), private monopoly is better than public monopoly. Our paper thus also relates to the literature on monopoly regulation, see Armstrong and Sappington (2007).

In this study of monopoly, we neglect many important aspects. Perhaps the most glaring omission is that this study presumes that the management of the monopoly, whether private or public, is fully rational and does not (try to) extract any private rents, such as shirking from work, taking bribes, using funds for luxurious offices, extravagant dinners, expensive but ill-motivated trips etc. Our focus is entirely on the monopolist's incentives to acquire information about consumer tastes. To the best of our knowledge, this is the first paper studying this aspect of monopoly behavior.

We begin by setting up the model in Section 2. In Section 3 we analyze the case of an unregulated private monopolist. Here, the monopolist strives to maximize its expected profit, defined as its revenue from sales, net of production costs and its costs for information acquisition about consumer preferences over product varieties (or locations). In Section 4 we consider the first-best case of a monopolist who strives to maximize expected social welfare, defined as the sum of its profit and consumer surplus, without any budget constraint. In Section 5 we consider a range of intermediate cases, between the previous two extremes, and obtain the second-best solution, that of a monopolist who strives to maximize expected social welfare under a budget constraint. All numerical results are shown in Section 6. Section 7 briefly discusses some potential extensions of the model and Section 8 concludes.

2 Model

Consider, thus, a monopolist in a market for a differentiated good sold in indivisible units. The monopolist has to choose a product variety $x \in X = \mathbb{R}$ and a price $p \in \mathbb{R}_+$

for this variety. The monopolist has a constant unit production cost $c \geq 0$. In this analysis, we abstract from fixed production costs. There is a continuum of consumers, each with unit demand for the good. Every consumer has Euclidean preferences over varieties, with a personal ideal product-variety $\theta \in X$. A consumer *type* is a pair $\tau = (\theta, v) \in X \times V$, where V is an interval. The utility for a consumer of type $\tau = (\theta, v)$ from buying one unit of product variety x at price p is

$$u = U_\tau (x, p) = v - p - (\theta - x)^2 \tag{1}$$

We will refer to $v \in V$ as the consumer type's *valuation* (of its ideal product variety). The utility from not buying is normalized to zero. A buyer of type τ buys a unit if and only if $U_\tau (x, p) \geq 0$.

Each consumer's type τ is his or her private information. The population is treated as a continuum with unit mass, and v and θ are treated as statistically independent random variables. Hence, there is no correlation between a consumer's valuation and his or her product ideal. The cumulative distribution function (CDF) for valuations, v, is denoted $F : V \to [0, 1]$, where F is absolutely continuous with density $f : V \to \mathbb{R}_+$ and such that the mean value $E[v]$ exists and exceeds the unit production cost c:

$$c < \bar{v} = \mathbb{E}[v] < +\infty$$

Each consumer's ideal product variety, θ, is the sum of a shared component, θ_0, common to all consumers in the population, and an idiosyncratic component (driven by fashion or social norms), ξ:

$$\theta = \theta_0 + \xi \tag{2}$$

These two components are statistically independent and normally distributed, both with mean-value zero. The shared component, θ_0, has variance $1/\alpha > 0$ and the idiosyncratic component, ξ, has variance $1/\beta > 0$. Aggregate demand for each product variety x is thus given by the demand function $D : \mathbb{R} \times \mathbb{R}_+ \to \mathbb{R}_+$ defined by

$$D(x, p) = \int_p^\infty \Pr\left[x - \sqrt{v - p} \leq \theta \leq x + \sqrt{v - p}\right] f(v)\, dv \tag{3}$$

The integrand represents the mass of willing consumers for each valuation v. This is illustrated in Figure 1 below. The area above the parabola is the set of "willing consumer types" when the price is at the horizontal dashed line and the supplied variety at the vertical dashed line. The monopolist's revenue from sales is thus its price multiplied by the population mass of willing consumers, those with types inside the parabola.

The monopolist knows all of the above. In addition, it can, at a cost, acquire more information about the shared component, θ_0, of consumer preferences over product

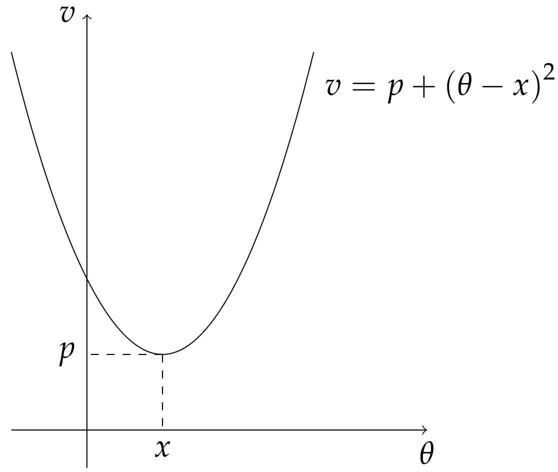

Figure 1: The set of willing buyers

varieties.[2] We represent such information as a signal

$$s = \theta_0 + \varepsilon$$

where the error (or noise) term ε is statistically independent of all other random variables and is normally distributed with mean-value zero and variance $1/q$, where the precision $q \geq 0$ is chosen by the monopolist, with $q = 0$ representing "no information acquisition" or "no signal". The cost for signal precision q is $C(q)$, where $C : \mathbb{R}_+ \to \mathbb{R}_+$ is twice differentiable with $C(0) = C'(0) = 0$, $C', C'' \geqslant 0$ and $C'(q) > 0$ for all $q > 0$.

The time-line is as follows: the monopolist first chooses its signal precision, then observes the signal realization s and thereafter chooses its product variety x and a price p. After these events, consumers observe the monopolist's product variety and price, sales occur and profits and utilities are realized. A *strategy* for the monopolist is a signal precision and, given any signal precision and any subsequently observed signal value, a product variety and price. Formally, a strategy is a pair $\langle q, \psi \rangle$, where $q \in \mathbb{R}_+$ and $\psi : \mathbb{R}_+ \times \mathbb{R} \to \mathbb{R} \times \mathbb{R}_+$ assigns to each precision-signal pair (q, s) some variety-price pair (x, p).

For any signal precision q, signal value s, product variety x and price p, let $\mathbb{E}_{(q,x,p)}[\Pi \mid s]$ be the monopolist's *posterior expected profit* (after the signal has been received), and let $\mathbb{E}_{(q,x,p)}[W \mid s]$ be the *posterior expected welfare*, where welfare is defined as consumer surplus plus monopoly profit. Let $\mathbb{E}_{(q,\psi)}[\Pi]$ and $\mathbb{E}_{(q,\psi)}[W]$ be the associated *ex ante* expected profit and welfare (before the signal is received) under strategy $\langle q, \psi \rangle$, defined by

$$\mathbb{E}_{(q,\psi)}[\Pi] = \mathbb{E}\left(\mathbb{E}_{(q,\psi_1(q,s),\psi_2(q,s))}[\Pi \mid s]\right)$$

[2] In order to keep focus on consumer preferences over varieties, we do not endogenize the monopolist's information about the consumer valuation distribution.

and

$$\mathbb{E}_{(q,\psi)}\left[W\right]=\mathbb{E}\left(\mathbb{E}_{(q,\psi_1(q,s),\psi_2(q,s))}\left[W\mid s\right]\right)$$

The monopolist's goal is to maximize a convex combination of these two latter expectations. Formally, it solves the optimization program

$$\max_{\langle q,\psi\rangle}\ (1-\gamma)\,\mathbb{E}_{(q,\psi)}\left[\Pi\right]+\gamma\mathbb{E}_{(q,\psi)}\left[W\right] \tag{4}$$

where $\gamma\in[0,1]$ is an exogenous parameter. At one end of the parameter spectrum, $\gamma=0$, we find the unregulated profit-maximizing monopolist while at the opposite end of the spectrum, $\gamma=1$, we find the welfare-maximizing monopolist, the monopolist that implements the first-best solution (in terms of welfare). As will be shown below, for an intermediate parameter value, $0<\gamma<1$, we will find the second-best public monopoly, that is a welfare-maximizing monopolist that faces the budget constraint that its *ex ante* expected profit be non-negative.

We note that optimality in Equation (4) requires that the monopolist's choice of product variety and price be optimal after (almost) all possible signal realizations.[3] We also note that, since we have normalized the consumer population to unity, all expected values above are bounded from below by $-c$ and from above by consumers' average valuation, $\bar{v}$, of their ideal product varieties minus the marginal cost of production. The latter claim follows from the observation that in the "best of worlds" production costs are nil and each consumer obtains his or her ideal product variety for free, in which case $E\left[W\right]=\bar{v}-c$. Hence, in general

$$\mathbb{E}_{(q,\psi)}\left[\Pi\right]\leq\mathbb{E}_{(q,\psi)}\left[W\right]\leq\bar{v}-c$$

We proceed to first analyze the case $\gamma=0$, then the case $\gamma=1$, and finally turn to all intermediate cases $\gamma\in(0,1)$. Henceforth, let Φ denote the CDF of the standard normal distribution $\mathcal{N}(0,1)$ and let ϕ be its density.

3 Unregulated profit maximization

We here consider an unregulated profit-maximizing monopolist; the case $\gamma=0$. Optimality in Equation (4) requires that the monopolist's choice of product variety and price be optimal after (almost) all possible signal realizations.

Suppose that the monopolist has chosen signal precision q and observed a signal value s, and is about to choose a product variety and price. In such a situation, the

[3] Since the signal by assumption has an absolutely continuous probability distribution, the goal function is unaffected by deviations from optimality on any subset of signal values with Lebesque measure zero.

monopolist will strive to maximize its conditionally expected profit,

$$\mathbb{E}_{(q,x,p)}\left[\Pi \mid s\right] = (p-c)\cdot \int_{p}^{\infty} \Pr\left[x-\sqrt{v-p} \leq \theta \leq x+\sqrt{v-p} \mid s\right] f(v)\, dv \\ - C(q) \tag{5}$$

This is the net earnings per unit sold, $p-c$, multiplied by the mass of willing consumers, minus the monopolist's information costs to obtain signal precision q. However, at this decision stage, the latter costs are sunk, so the last term is irrelevant for the monopolist's choice of product variety and price. The following result establishes that the monopolist's choice of product variety is a random variable that depends linearly on the signal, while its price is deterministic and independent of the signal value. Moreover, we find that the choice of product variety is more sensitive to the signal the higher its precision. In particular, if the monopolist has (previously) chosen to acquire no information, $q=0$, then it will opt for product variety $x=0$, the *ex ante* expected mean value of consumers' ideal product variety.

PROPOSITION 1. *For any signal precision $q \geq 0$ and any observed signal value $s \in \mathbb{R}$, the profit-maximizing monopolist ($\gamma = 0$) will choose product variety*

$$x^* = \frac{q}{\alpha+q}\cdot s \tag{6}$$

and set its price p^* so that

$$p^* \in \arg\max_{p\geq 0}\ (p-c)\cdot \int_{p}^{\infty}\left[\Phi\left(\sqrt{\frac{(\alpha+q)(v-p)\beta}{\alpha+\beta+q}}\right)-\frac{1}{2}\right] f(v)\, dv \tag{7}$$

Moreover, the set on the right-hand side of Equation (7) is a non-empty and compact subset of the open interval $(c,+\infty)$.

Proof: Conditional upon an observed signal value s, the monopolist's posterior for θ_0 is normally distributed,

$$\theta_0\,|s \sim N\left(\frac{qs}{\alpha+q}, \frac{1}{\alpha+q}\right)$$

By assumption, the idiosyncratic taste parameter ξ is statistically independent of both θ_0 and s, so $\xi\,|s \sim \xi \sim N(0, 1/\beta)$ and thus

$$\theta\,|s \sim N\left(\frac{qs}{\alpha+q}, \frac{1}{\alpha+q}+\frac{1}{\beta}\right)$$

The conditional random variable $\theta\,|s$ is symmetrically and unimodally distributed around its mean-value, $\mu = sq/\left(\alpha+q\right)$, so for any $p \geq 0$, the monopolist will choose a product variety x such that, for any valuation $v \in V$, the associated interval of "willing buyer types", $\theta \in [x-\sqrt{v-p}, x+\sqrt{v-p}]$, is centered on μ. Hence, it is optimal for the monopolist to select product variety x^* as given in Equation (6). Given this choice of product variety, and conditional upon the realized signal s, the monopolist chooses its price p so as to maximize

$$\frac{p-c}{\sigma_\theta\sqrt{2\pi}} \cdot \int_p^\infty \left(\int_{\mu-\sqrt{v-p}}^{\mu+\sqrt{v-p}} \exp\left[-\frac{1}{2}\left(\frac{t-\mu}{\sigma_\theta}\right)^2\right] dt \right) f\left(v\right) dv$$

where $\sigma_\theta^2 = \left(\alpha+\beta+q\right)/\left[\left(\alpha+q\right)\beta\right]$ is the variance of $\theta\,|s$. After a change of variables one obtains Equation (7). The maximand in Equation (7) is a continuous function of p that vanishes at $p = c$ and as $p \to +\infty$.[4] Moreover, the maximand is positive at intermediate values of p. Hence, by Weierstrass' maximum theorem, the set of maximizers is a non-empty and compact subset of the open set V. Q.E.D.

Hence, the monopolist's optimal variety/price strategy $\psi^* = (\psi_1^*, \psi_2^*)$ satisfies $\psi_1^*\left(q,s\right) = qs/\left(\alpha+q\right)$ for all $q \geq 0$ and all $s \in \mathbb{R}$, and ψ_2^* is constant across signal values, $\psi_2^*\left(q,s\right) = \psi_2^*\left(q,s'\right)$ for all $q \geq 0$ and $s, s' \in \mathbb{R}$. We will henceforth thus write $p^*\left(q\right)$ for $\psi_2^*\left(q,s\right)$. Since every optimal price is interior (with respect to the support of the valuation distribution), the price p has to satisfy the first-order condition that the derivative of the maximand in Equation (7) with respect to p, is zero. This condition in fact uniquely determines the pricing strategy $p^* : \mathbb{R}_+ \to \mathbb{R}_+$, to be specified in the next result. Let

$$\eta\left(q\right) = \sqrt{\frac{\left(\alpha+q\right)\beta}{\alpha+\beta+q}} \tag{8}$$

This is the square root of the precision of the posterior (after the signal) estimate of individuals' ideal product varieties.

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[4]To see that the maximand tends to zero as p tends to plus infinity, write the maximand as $m\left(p\right)$ and note that

$$m\left(p\right) \leq p\int_p^{+\infty} g\left(v\right) dv \leq \int_p^{+\infty} vg\left(v\right) dv$$

Since $\bar{v} < +\infty$, the last integral has to converge to zero as $p \to +\infty$. To see the latter claim, note that

$$\bar{v} = \int_0^p vg\left(v\right) dv + \int_p^{+\infty} vg\left(v\right) dv \quad \forall p > 0$$

where the first integral converges to $\bar{v}$ as $p \to +\infty$.

PROPOSITION 2. *For every $q \geq 0$ the monopolist's price, $p = p^*(q)$, is the unique solution of the equation*

$$\int_{p}^{\infty} \left[\Phi\left(\eta(q)\sqrt{v-p}\right) - \frac{1}{2} - \frac{(p-c)\,\eta(q)}{2\sqrt{v-p}} \phi\left(\eta(q)\sqrt{v-p}\right) \right] f(v)\, dv = 0 \tag{9}$$

Moreover, the solution lies in the open interval $(c, +\infty)$.

Proof: The left-hand side of Equation (9) is the derivative of the maximand in Equation (7) with respect to p. Writing $k(p)$ for the left-hand side in Equation (9), note that $k : [c, +\infty) \to \mathbb{R}$ is continuous and satisfies $k(c) > 0$. Moreover, the integrand in Equation (9) is point-wise decreasing in p, for any given $v > c$, and the interval of integration is shrinking, so k is strictly decreasing. This establishes the claimed uniqueness. Existence follow from Proposition 1 and the fact that every profit-maximizing price is interior and hence necessarily meets the first-order condition in Equation (9). **Q.E.D.**

REMARK 1. *Equation (9) determines the optimal price by comparing the benefit and the cost of increasing price; the optimal price is determined by the condition that there exits no gains by changing price. Figure 2 shows us the effect of increasing the price from p to $p + \Delta p$, where Δp is positive and small increment. The effects can be decomposed into two parts. For those consumers who have valuations below $p + \Delta p$, the increase of the price causes them to exit the market (the shaded area in Figure 2). This decreases the profit by*

$$(p-c) \int_{p}^{p+\Delta p} \left[2\Phi\left(\eta(q)\sqrt{v-p}\right) - 1 \right] f(v)\, dv$$

For those who have valuation $\tilde{v}$ above $p + \Delta p$, some will stay in the market and others will exit. This shifts the sales revenues of the monopolist from the area of the rectangle with solid sides to the area of the rectangle with dashed sides, a decrease in the rectangle's width and an increase of its height. Furthermore, the decrease in width is symmetric around the mean of the monopolist's posterior estimate of consumers' ideals. The optimal price satisfies the first-order condition that any change from p^* to $p^* + \Delta p$ or $p^* - \Delta p$ would decrease the monopoly's revenue.

Equation (9) shows that the monopolist sets the price according to square root of the precision of the posterior estimate of consumers' ideals. Inspired by Figure 2, we see that the effect of $\eta(q)$ on the optimal price is affected by consumers' valuation distribution. To see this, note that increases in $\eta(q)$ make the monopolists believe more consumers are concentrated around x; so it is more beneficial to shift the revenue from the rectangle with solid sides to the rectangle with dashed sides by increasing the price from p to $p + \Delta p$. However, higher $\eta(q)$ also implies that it is more costly to give up the shaded area in Figure 2. If consumers' valuation

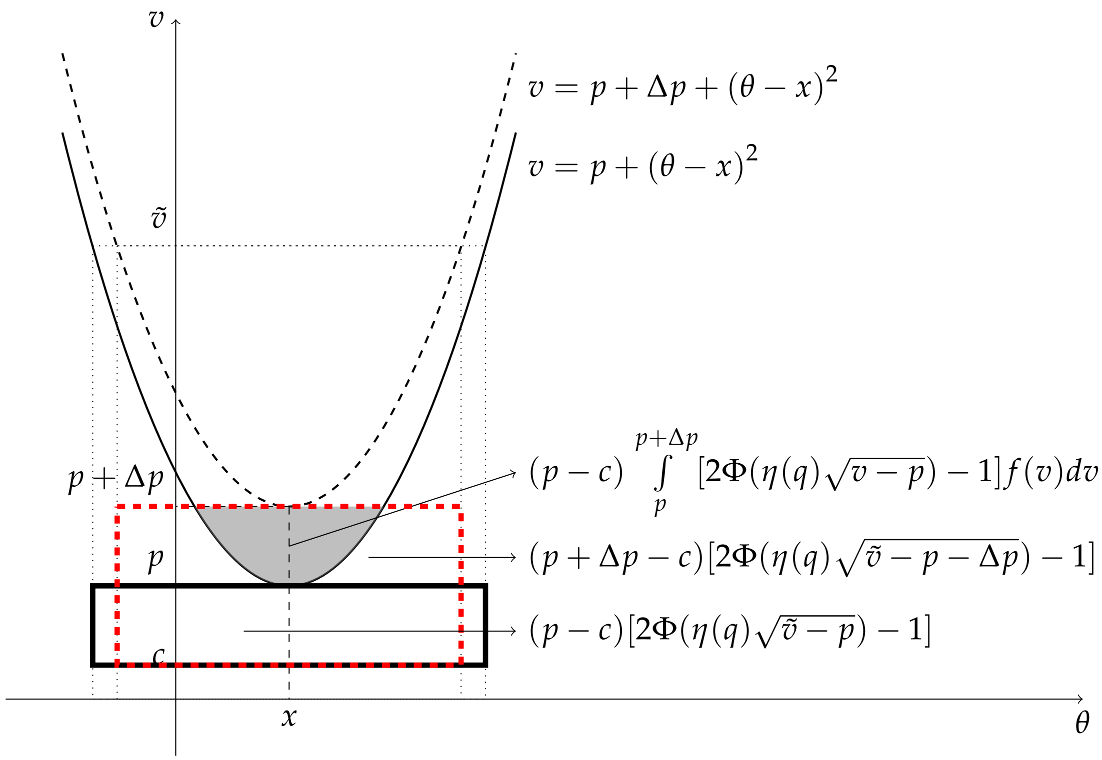


Figure 2: The effect of increasing price

is very concentrated around p in Figure 2, it is very costly for the monopolist to give up the shaded area; in this case, increases in $\eta(q)$ force the monopolist to decrease the price. On the contrary, if consumers valuation is very concentrated above $p + \Delta p$, it is very cheap for monopolist to give up the shaded area and it is very beneficial for the monopolist to increase the price; in this case, increases in $\eta(q)$ force the monopolist to increase the price. Therefore, the effect of $\eta(q)$ on the price is determined by the location of the price and the consumer's valuation distribution. Since the price is endogenously determined by $\eta(q)$, depending to the consumers' valuation distribution, $\eta(q)$ may have a non-monotone effect on the price.

If all consumers have the same valuation $\bar{v}$, the shaded area in Figure 2 vanishes. Therefore, an increase of $\eta(q)$ forces the monopolist to increase the price. Similarly, if consumers' valuations are uniformly distributed on an interval $[v_{\min}, v_{\max}]$, the monopolists value of the shaded area in Figure 2 tends to zero as Δp goes to zero. Hence, increases in $\eta(q)$ will still force the monopolist to increase the price.

Furthermore, from Figure 2, we know that the value of the shaded area is monotonically decreasing in the unit production cost c; at the same time, the benefit of shifting from the rectangle with solid sides to the rectangle with dashed sides is

$$\Delta p \left[2\Phi \left(\eta(q) \sqrt{\tilde{v} - p - \Delta p} \right) - 1 \right]$$

$$-2\,(p-c)\int_{-\sqrt{\bar{v}-p}}^{-\sqrt{\bar{v}-p-\Delta p}}\frac{\eta\,(q)}{\sqrt{2\pi}}\exp\left[-\frac{1}{2}\eta\,(q)^2\,t^2\right]dt$$

This is obviously monotonically increasing in c. Therefore, given the posterior estimate precision $\eta\,(q)$ and consumers' valuation distribution, increases in the unit production cost c force the monopolist to increase the price.

We are finally in a position to consider the monopolist's choice of signal precision, or, in other words, how well-informed it wants to be about consumer preferences. The monopolist will evidently never choose a signal precision so high that its cost cannot be recovered by its sales. Since sales revenues cannot exceed consumers' mean valuation, $\bar{v}$, and since information costs are non-negative and strictly increasing, Weierstrass' maximum theorem implies that the the set Q^* of the monopolist's optimal signal-precisions $q \geq 0$ is non-empty and compact and can be characterized as follows:

$$Q^* \;=\; \arg\max_{q\in Q}\left(\begin{array}{l}\left[p^*\,(q)-c\right]\displaystyle\int_{p^*(q)}^{\infty}\left[2\Phi\left(\eta\,(q)\,\sqrt{v-p^*\,(q)}\right)-1\right]f\,(v)\,dv\\ -\,C\,(q)\end{array}\right) \tag{10}$$

where $Q=\left[0,C^{-1}\,(\bar{v})\right]$. Moreover, since the marginal cost of information acquisition by assumption is zero at zero precision, any optimal signal precision must be positive. In sum:

PROPOSITION 3. *The set Q^* is a non-empty and compact subset of the open interval $\left(0,C^{-1}\,(\bar{v})\right)$. If $q^*\in Q^*$, then $q=q^*$ satisfies the first-order condition*

$$(p-c)\int_{p}^{\infty}\sqrt{v-p}\,\phi\left(\eta\,(q)\,\sqrt{v-p}\right)f\,(v)\,dv \;=\; \eta\,(q)\left(\frac{\alpha+\beta+q}{\beta}\right)^2 C'\,(q) \tag{11}$$

evaluated at $p=p^*\,(q)$.

In sum, then, the monopolist's decision problem has a solution, and once a signal precision $q^*\in Q^*$ has been found, Equation (9) will produce a unique monopoly price $p=p^*\,(q^*)$ associated with that signal quality.

4 First best

We now turn to the first-best case, $\gamma=1$, that of a monopolist striving to maximize welfare, but otherwise is identical with a private monopoly. In particular, the monopolist faces the same informational constraints and costs etc. as before. Moreover, this public monopolist does not face any budget constraint; it is as if its costs can be covered by a lump-sum tax on all individuals in the consumer population, irrespective of whether or not they actually consume the good or not. As we will see, this type of monopolist will set the price of the good at unit production cost and raise all funds by way of a lump-sum tax.

Just as in the case of private monopoly, we solve the monopolists' decision problem by backward induction. Suppose, thus, that the monopolist has already chosen its signal quality $q \geq 0$ and observed the signal s. For any product variety x and price p she may choose, the resulting expected welfare, conditional upon $S = s$, is

$$\mathbb{E}_{(q,x,p)}\left[W|s\right] = \int_{p}^{\infty}\left(\int_{x-\sqrt{v-p}}^{x+\sqrt{v-p}}\left[v-(\theta-x)^{2}-c\right]dH\left(\theta|s\right)\right)f\left(v\right)dv - C\left(q\right) \qquad (12)$$

where H is the CDF of the conditional random variable $\theta|s$ that represents consumers' preferences over product varieties (given the observed signal value s). It is not difficult to verify that a welfare-maximizing monopoly, once it has chosen its signal precision and observed its signal, will choose the same product variety as does a profit-maximizing monopolist, but will set its price equal to unit production cost.

PROPOSITION 4. *For any given signal precision $q \geq 0$ and any observed signal value $s \in \mathbb{R}$, the welfare-maximizing monopolist ($\gamma = 1$) will choose product variety*

$$\hat{x} = x^{*} = \frac{q}{\alpha+q}\cdot s$$

and sets price $\hat{p} = c$.

Proof: As noted in the proof of Proposition 1,

$$\theta\,|s \sim N\left(\frac{qs}{\alpha+q}, \frac{1}{\alpha+q}+\frac{1}{\beta}\right).$$

The conditional random variable $\theta\,|s$ is thus symmetrically and unimodally distributed around its mean-value, $sq/\left(\alpha+q\right)$. For any p and x, and any valuation v, the associated interval of "willing" buyer types θ is $[x-\sqrt{v-p}, x+\sqrt{v-p}]$, centered on x. Moreover, for any v, the integrand in Equation (12), $v-(\theta-x)^{2}-c$, is positive on the interval of willing buyer types, granted $c < p$, and it is strictly concave in θ with maximum at $\theta = x$. Hence, for any $p > c$ and any $v \in V$, the inner integral,

$$\int_{x-\sqrt{v-p}}^{x+\sqrt{v-p}}\left[v-(\theta-x)^{2}-c\right]dH\left(\theta|s\right),$$

is maximized when $x = x^*$. Given this choice of product variety, given $c < p$, and still conditional upon the realized signal s, expected welfare can be written as

$$\mathbb{E}_{(q,x,p)}\left[W|s\right] = \int_{p}^{\infty}\left(\int_{sq/(\alpha+q)-\sqrt{v-p}}^{sq/(\alpha+q)+\sqrt{v-p}}\left[v-\left(\theta-sq/\left(\alpha+q\right)\right)^2-c\right]dH\left(\theta|s\right)\right) f\left(v\right)dv - C\left(q\right) \tag{13}$$

Consider the inner integral,

$$\int_{sq/(\alpha+q)-\sqrt{v-p}}^{sq/(\alpha+q)+\sqrt{v-p}}\left[v-\left(\theta-sq/\left(\alpha+q\right)\right)^2-c\right]dH\left(\theta|s\right)$$

for any given $v \in V$. It is positive and decreasing in p for all $p \in (c, v)$. For $p \leq c$ it negative and increasing in p (by Leibnitz' rule or inspection of Figure 1). Moreover, the outer integration interval in Equation (12), $(p, +\infty)$, is shrinking in p. Hence $\mathbb{E}_{(q,x,p)}\left[W|s\right]$ is maximized at $p = c$. **Q.E.D.**

The intuition behind the difference between the private monopolist's and the public monopolist's pricing is simple; for the public monopolist a higher price has no benefit, since any potential rise in its profits would be matched by an equally large reduction in consumer surplus. Hence, a higher price is only potentially harmful for the unconstrained welfare maximizing monopolist. In effect, it covers its costs by a lump-sum tax on all consumers. Moreover, the monopolist does not want to produce for the consumers whose valuation is less than the marginal cost of production since those consumption decreases the social welfare.

Having solved for the public monopolist's choice of product variety and price, we are now in a position to analyze its choice of signal precision. Using Proposition Proposition 4, one immediately obtains the following expression for the *ex ante* expected welfare (as expected before the signal has been observed):

$$\mathbb{E}_{(q,\hat{\psi})}\left[W\right] = \mathbb{E}\left[\int_{c}^{\infty}\left(\int_{sq/(\alpha+q)-\sqrt{v-c}}^{sq/(\alpha+q)+\sqrt{v-c}}\left[v-\left(\theta-sq/\left(\alpha+q\right)\right)^2-c\right]dH\left(\theta|s\right)\right) f\left(v\right)dv\right] - C\left(q\right)$$

where $\hat{\psi}$ is the optimal decision-rule defined in Proposition 4 and the expectation on the right-hand side is taken over the (normally distributed) signal. By a simple change of variables, we obtain the simpler expression

$$\mathbb{E}_{(q,\hat{\psi})}[W] = \int_{c}^{\infty}\left(\int_{-\sqrt{v-c}}^{\sqrt{v-c}} 2t\Phi\left[\eta(q)\,t\right]dt\right) f(v)\,dv - C(q) \tag{14}$$

where $\eta(q)$ is defined in Equation (8). The double integral in Equation (14) is differentiable and strictly increasing in $\eta(q)$. Hence this term is strictly increasing and differentiable in q. Let $\hat{Q} \subseteq \left[0, C^{-1}(\bar{v})\right]$ denote the set of socially efficient signal precisions. The Weierstrass maximum theorem guarantees that there exists at least one such signal precision, and, in fact, that this set is compact.

PROPOSITION 5. *The set $\hat{Q}$ is non-empty and compact. If $\hat{q} \in \hat{Q}$, then either $\hat{q} = 0$ or $\hat{q} = q$ for some $q > 0$ that satisfies*

$$\int_{c}^{\infty}\left(\int_{-\sqrt{v-c}}^{\sqrt{v-c}} t^2\phi\left(\eta(q)\,t\right)dt\right) f(v)\,dv = \left(\frac{\alpha+\beta+q}{\beta}\right)^2 \eta(q)\,C'(q) \tag{15}$$

Clearly, the welfare-maximizing monopolist's signal precision $\hat{q}$ is positive if the marginal cost of information-acquisition at zero precision is zero, which is the case on our parametric specifications in the numerical simulations.

5 Intermediate cases

We now turn to the canonical intermediate case when $\gamma \in (0,1)$. This case thus spans from private monopoly ($\gamma = 0$, Section 3) to first-best public monopoly ($\gamma = 1$, Section 4). As will be seen, we obtain the solutions for all public monopolies that face any exogenous budget constraint. This setting is in line with the classical Ramsey-Boiteux approach, see Ramsey (1927) and Boiteux (1971). More precisely, the monopolist here strives to maximize expected welfare, under the constraint that its expected profit reaches a pre-specified level, to be denoted B; the arguably most relevant case being that of self-financing, that is, $B = 0$. But first we solve Equation (4) for any given $\gamma \in (0,1)$.

Like in the two preceding cases, we solve this monopolist's decision problem by backward induction. We have already shown that for any given signal precision q and price p, and after observing the signal s, welfare and profits are both maximized at product variety $x = sq/(\alpha+q)$. The following result is thus a direct application of those conclusions.

COROLLARY 1. *For any given signal precision $q \geq 0$ and any observed signal value $s \in \mathbb{R}$, the optimal product variety for the generalized monopoly ($\gamma \in (0,1)$) is*

$$x^{**} = x^{*} = \hat{x} = \frac{q}{\alpha+q}\cdot s.$$

Given the optimal product variety (a random variable), the social welfare conditional on signal s is given by

$$\mathbb{E}_{(q,\psi_1^{**},p)}[W|s] = \int_p^\infty \left(\int_{-\sqrt{v-p}}^{\sqrt{v-p}} 2t\Phi(\eta(q)t)\,dt \right) f(v)\,dv + (p-c)\int_p^\infty \left[2\Phi\left(\eta(q)\sqrt{v-p}\right)-1\right] f(v)\,dv - C(q)$$

where the first term is consumer surplus, conditional on the signal, and the last two terms together make up the profit of the monopolist (and these do not depend on the signal). For any signal quality q that the monopolist may have chosen, it should thus set its price p^{**} so that solves

$$\max_{p\in V}\left[\gamma\cdot\int_p^\infty \left(\int_{-\sqrt{v-p}}^{\sqrt{v-p}} 2t\Phi(\eta(q)t)\,dt \right) f(v)\,dv + (p-c)\int_p^\infty \left[2\Phi\left(\eta(q)\sqrt{v-p}\right)-1\right] f(v)\,dv - C(q)\right] \tag{16}$$

Taking derivative of the maximand in Equation (16), we obtain a characterization of the optimal price.

PROPOSITION 6. *Given any $q \geq 0$ and $\gamma \in (0,1)$, the monopolist's optimal price p^{**} is the unique solution of the equation*

$$\int_p^\infty \left[(1-\gamma)\left[2\Phi\left(\eta(q)\sqrt{v-p}\right)-1\right] - \frac{(p-c)\,\eta(q)}{\sqrt{v-p}}\phi\left(\eta(q)\sqrt{v-p}\right)\right]\cdot f(v)\,dv = 0 \tag{17}$$

Proof: To prove the uniqueness of the solution, note that for each $v \in V$, the integrand in Equation (17) is continuous and monotonically decreasing on the interval $[c, v)$, from a positive value at $p = c$ towards minus infinity as $p \to v$. As shown in the proof of Proposition Proposition 2, this property is inherited by the integral in Equation (17). **Q.E.D.**

In sum, then, our qualitative result for the strategy ψ^* of the profit-maximi-zing monopolist holds for all $\gamma \in [0,1]$: the monopolist's optimal variety/price strategy ψ always satisfies $\psi_1(q,s) = qs/(\alpha+q)$ for all $q \geq 0$ and all $s \in \mathbb{R}$, and ψ_2 is always constant across signal values.

Equation (17) defines the optimal pricing rule ψ_2^{**}, which we, with a slight abuse of notation, write as $p = p^{**}(q)$. The marginal benefit of increasing price for the monopolist is a weighted sum of the benefit to social welfare and the benefit to the monopolist's profit. As we have seen, increasing the price generally decreases welfare, so the marginal benefit of price to the monopolist in this intermediate case is smaller than for the pure profit-maximizing monopolist, who places weight $\gamma = 0$ on consumer surplus. This implies that, given any signal precision q, the intermediate monopolist will charge a lower price than the profit-maximizing monopolist.

The monopolist should choose its signal precision q^{**} so that it solves

$$\max_{q\in\left[0,C^{-1}(\bar{v})\right]} \left[\gamma \cdot \int_{p^{**}(q)}^{\infty} \left(\int_{-\sqrt{v-p^{**}(q)}}^{\sqrt{v-p^{**}(q)}} 2t\Phi\left(\eta\left(q\right)t\right)dt \right) f\left(v\right)dv \right.$$
$$\left. + \; \left[p^{**}\left(q\right) - c\right] \cdot \int_{p^{**}(q)}^{\infty} \left[2\Phi\left(\eta\left(q\right)\sqrt{v - p^{**}\left(q\right)}\right) - 1\right] f\left(v\right)dv \; - \; C\left(q\right) \right] \tag{18}$$

Applying the envelope theorem and differentiating, we obtain a characterization of the optimal signal precision for any given $\gamma \in (0,1)$:

PROPOSITION 7. *Given $\gamma \in (0,1)$, the monopolist's signal precision is either $q^{**} = 0$ or $q^{**} = q$ for some $q > 0$ that satisfies the equation*

$$\gamma \int_{p}^{\infty} \left(\int_{-\sqrt{v-p}}^{\sqrt{v-p}} t^2\phi\left(\eta\left(q\right)t\right)dt \right) f\left(v\right)dv \; + \; (p-c) \cdot$$
$$\int_{p}^{\infty} \left[\phi\left(\eta\left(q\right)\sqrt{v-p}\right)\sqrt{v-p}\right] f\left(v\right)dv \; = \; \left(\frac{\alpha+\beta+q}{\beta}\right)^2 \eta\left(q\right)C'\left(q\right) \tag{19}$$

The marginal benefit of private information acquisition is also a weighted sum of the benefit to social welfare and the benefit to the private monopolist's profit. The analysis before shows us that the comparison between the marginal benefit to social welfare and the marginal benefit to the firm's profit depends on the spread of the consumers' valuation. Therefore, the inefficiency of the profit maximizing monopolist's level of information, as compared with the first-best information level, is arguably mainly determined by this spread.

5.1 Second-best: budget-constrained welfare maximization

We now turn to the case of a budget-constrained welfare-maximizing monopolist, or, more precisely, a monopolist who chooses its strategy $\langle q, \psi \rangle$ so as to maximize expected welfare, $\mathbb{E}_{(q,\psi)}[W]$, subject to the constraint that its expected profit, $\mathbb{E}_{(q,\psi)}[\Pi]$, is at least B, where $B \geq 0$ is exogenously given. The case $B = 0$ is of particular interest, since it represents a familiar second-best situation, that of a monopolist who strives to maximize welfare under the requirement that it should cover its costs by its own revenues.

Let $\pi(0)$ be the profit obtained by the monopolist in Section 3 ($\gamma = 0$), let $\pi(1)$ be the profit obtained by the monopolist in Section 4 ($\gamma = 1$), and for each $\gamma \in (0, 1)$, let $\pi(\gamma)$ be the profit obtained by the monopolist in the present section. We then have $\pi(0) > 0 > \pi(1)$, and $\pi(\gamma)$ is continuous in γ for $\gamma \in [0, 1]$. (The last claim follows from Berge's maximum theorem.) Hence, by continuity there exists at least one $\gamma \in (0, 1)$ for which $\pi(\gamma) = 0$.

PROPOSITION 8. *Let γ_o be such that $\pi(\gamma_o) = 0$. A monopolist that maximizes $\mathbb{E}_{(q,\psi)}[W]$ subject to the budget constraint $\mathbb{E}_{(q,\psi)}[\Pi] \geq 0$ will choose the same strategy $\langle q^{**}, \psi^{**} \rangle$ as defined above, for $\gamma = \gamma_o$, and the budget constraint will be precisely met.*

That is to say, we find the budget-constrained monopolist's behavior by picking the parameter value γ so that the budget constraint is exactly met. The intuition behind this result is simple: the Lagrangian associated with the budget-constrained monopolist's decision problem can be written as

$$L(q, \psi, \lambda) = \mathbb{E}_{(q,\psi)}[W] + \lambda \mathbb{E}_{(q,\psi)}[\Pi],$$

where $\lambda \geq 0$ is the Lagrangian multiplier associated with the budget constraint. By setting $\lambda = \lambda_o = (1 - \gamma_o) / \gamma_o$, maximization of $L(q, \psi, \lambda_o)$, given λ_o, is identical with solving our intermediate monopolist's problem when $\gamma = \gamma_o$. And by definition of this particular γ-value, $\mathbb{E}_{(q,\psi)}[\Pi] = 0$, so the budget-constraint is met. To see the intuition why the budget constraint is necessarily precisely met, suppose, by contradiction, that the constraint were not binding, so that the monopolist now earns a profit above $B = 0$. Then, given its signal precision q^{**}, the monopolist could reduce its price slightly without violating the budget constraint (by continuity), and this way obtain a slightly higher social welfare, since the latter is monotonically decreasing in the price, as shown in the proof of Proposition 4.[5]

6 Numerical simulations

In order to illustrate the above results, we will now consider numerical simulations results for the special case of a log-normal value distribution and information costs that are a positive power of the signal precision. As will be seen, we will be able to numerically identify the monopolist's unique signal precision and price, and make comparative-statics experiments with these.

[5] Actually for any B less than the profit of the unregulated private monopoly and more than the profit of public monopoly, there is one γ_B such that $\pi(\gamma_B) = B$, the budget constraint will be precisely met.

6.1 Profit maximization

We now turn to our numerical simulations results. Assume that $C(q) = bq^a$ for some $b > 0$ and $a \geq 1$. Furthermore, the consumers' valuation is assumed to be log-normally distributed with mean $\bar{v}$ and standard deviation σ.[6]

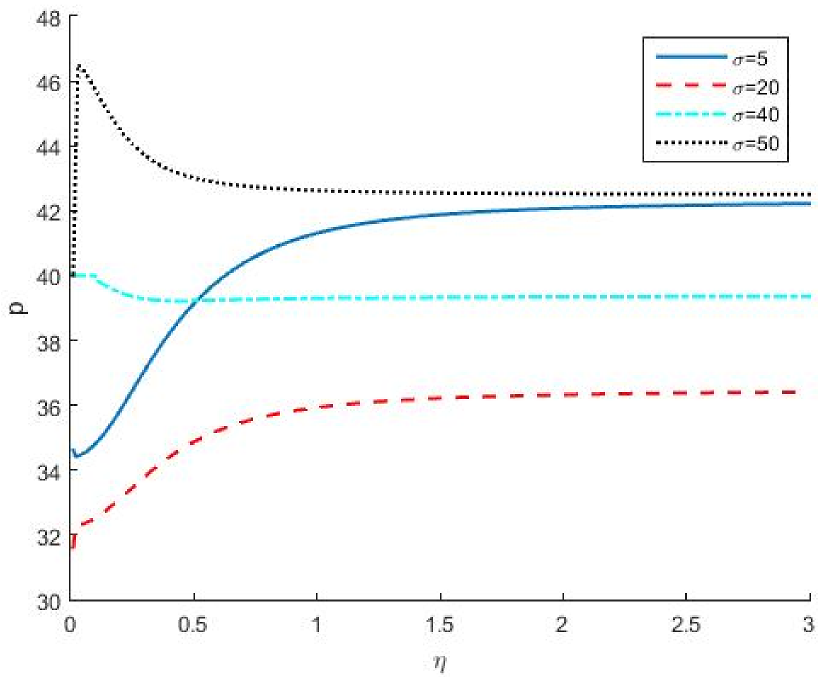


Figure 3: Equilibrium price and posterior estimate precision

The first batch of our numerical simulations studies the effect of $\eta(q)$ on the monopolist's optimal price (that is, it only studies Equation (9)). Figure 3 shows us the numerical result when $c = 5$ and $\bar{v} = 50$. The result confirms our conjecture that, depending on the valuation distribution, the effect of $\eta(q)$ on the monopolist's optimal price may be either monotone or non-monotone. When the variance of consumers' valuation distribution is small enough, higher $\eta(q)$ monotonically increases the price; when the variance is large enough, $\eta(q)$ has a non-monotone effect on the optimal price. This result is consistent with our reasoning above and the property of log-normal distribution. With very small σ (such as $\sigma = 5$ or $\sigma = 20$), the mode of the log-normal distribution is around $\bar{v}$ so that a large number of consumers' valuation is concentrated around $\bar{v}$;[7] however, the price is much smaller than $\bar{v}$. According to our analysis above, higher $\eta(q)$ will increase the price. As σ increases, the mode de-

[6] In this case, denoted the consumers' valuation by v, then $\ln(v)$ is normally distributed with mean $\ln \bar{v} - \ln \sqrt{1 + \frac{\sigma^2}{\bar{v}^2}}$ and variance $\ln\left(1 + \frac{\sigma^2}{\bar{v}^2}\right)$.

[7] Note that the mode of the log-normal distribution is $\bar{v} \exp\left(-\frac{3}{2} \ln\left(1 + \sigma^2/\bar{v}^2\right)\right)$.

creases monotonically; when the mode of the distribution is around the price (when $\sigma = 40$), increases in $\eta(q)$ force the monopolist to decrease the price. When σ is large enough such that the mode is much smaller than the price, increases in $\eta(q)$ will firstly force the monopolist to increase the price and then force the monopolist to decrease the price.

The second batch of our numerical simulations is focused on comparative statics when consumers' valuation distribution is given. During the second batch, we assume that consumers' valuation is log-normally distributed with mean $\bar{v} = 50$ and standard deviation $\sigma = 50$.

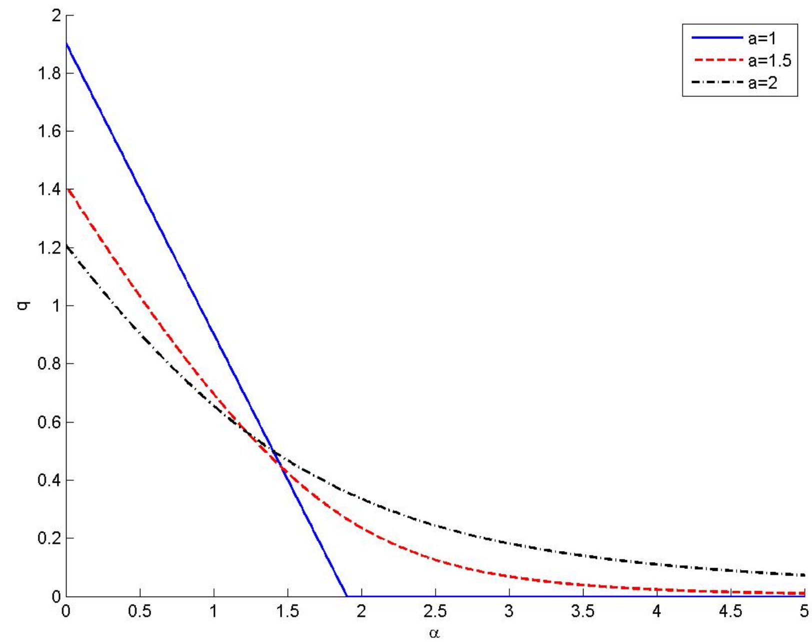
Figures 4a and 4b show the optimal information acquisition and optimal price as functions of α when $a = 2$, $b = 0.1$, $c = 5$ and $\beta = 1$. Figure 4a shows that a more precise prior for the shared component crowds out the monopolist's incentive for information acquisition. Furthermore, with linear information acquisition cost function, when α is small enough, the private information acquisition and α are perfect substitutes. Furthermore, the information acquisition decreases in a lower speed than the increase of α such that the monopolist has a more precise posterior estimate of the consumers' ideal products. With our parameter, this decreases the monopolist's price in equilibrium, which is consistent with the results from Figure 3.

Figures 5a and 5b show the numerical solutions as functions of β when $a = 2$, $b = 0.1$, $c = 5$ and $\alpha = 1$. Figure 5a shows that the more concentrated consumers' idiosyncratic ideal product varieties are, the more private information the monopolist will acquire. The intuition behind the result is that, with more concentrated ideal product varieties, private information is more valuable since it helps estimate more consumers' ideal products. Therefore, large concentration in consumers' ideal products increase the monopolists' precision of posterior estimate of consumers' ideal products; to be consistent with Figure 3, this will decrease the monopolist's price in equilibrium.

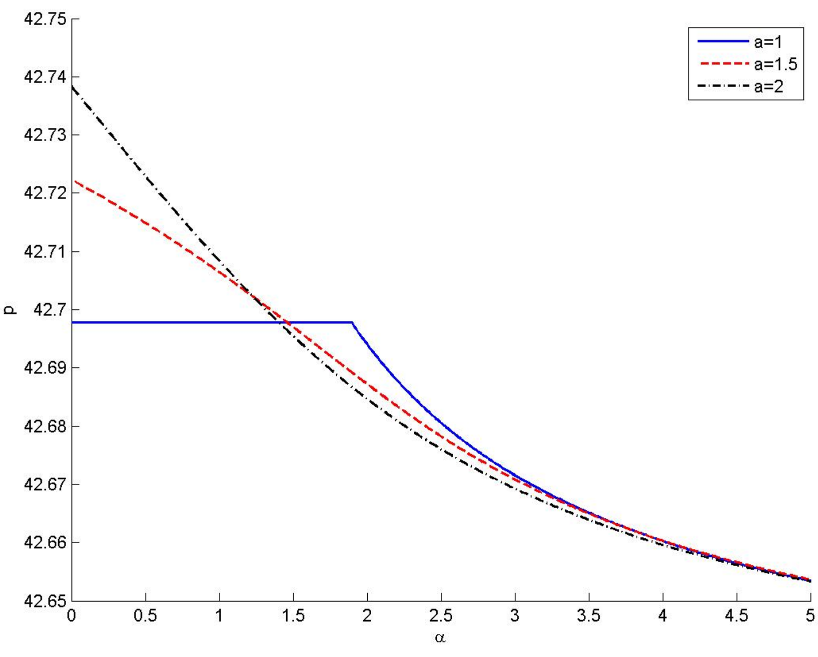
Figures 6a and 6b show the numerical solutions as functions of the power a in the cost function when $\alpha = \beta = 1$, $c = 5$ and $b = 0.1$. The effect of a on information acquisition is non-monotone, and is affected by other parameters, especially by the cost parameter b.[8] An increase in a reduces the marginal information acquisition cost for signal precision q less than $\exp\left(-\frac{1}{a}\right)$ and increases the marginal information acquisition cost for q greater than $\exp\left(-\frac{1}{a}\right)$.[9] When both a and b are quite small, the signal precision in equilibrium is very high; in this case, an increase in a decreases the monopolists information acquisition. As a increases, the information acquisition falls

[8] An increase in b monotonically increases the marginal cost of information acquisition, and therefore monotonically decreases the information acquisition in equilibrium.

[9] To see this, taking derivative of the marginal information acquisition cost with respect to b, we have $bq^{a-1}(a \ln q + 1)$.

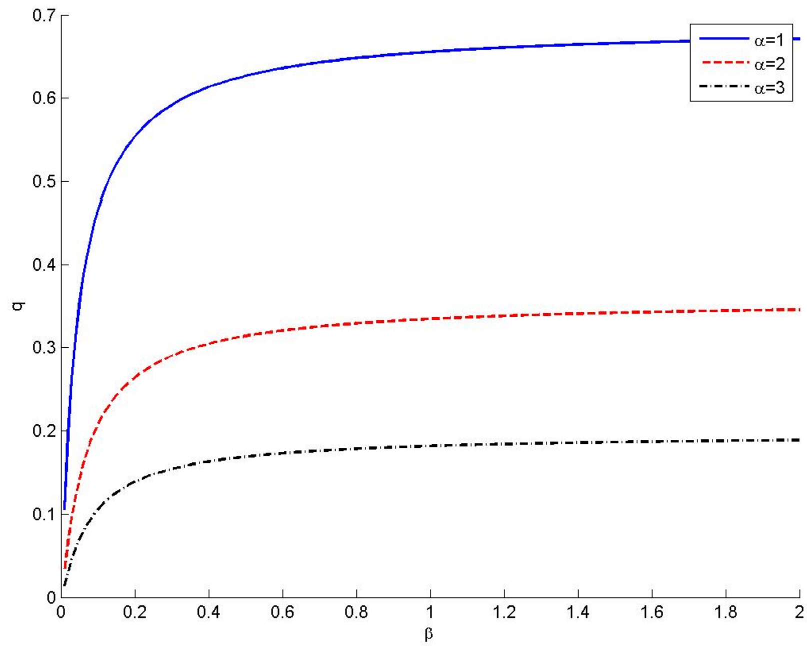


(a) Information acquisition

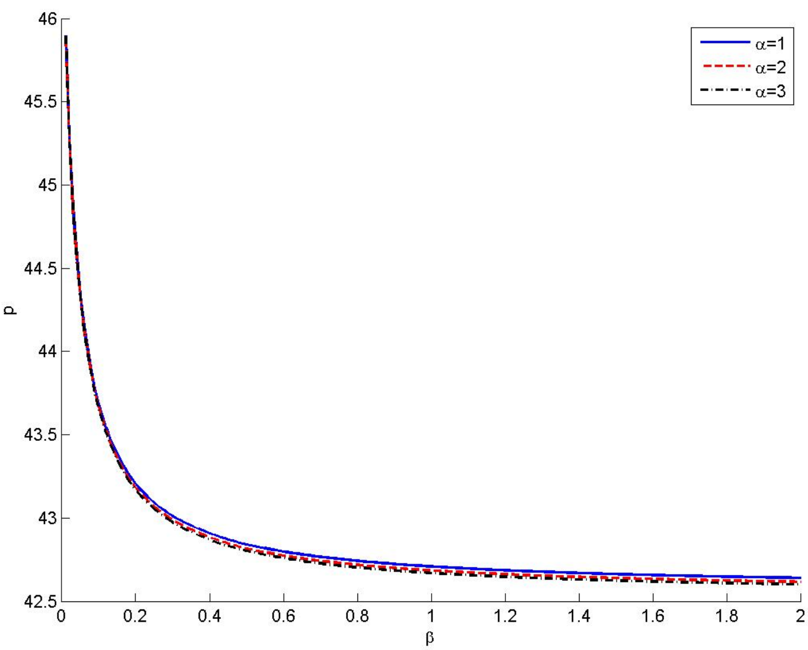


(b) Price

Figure 4: The effect of shared component precision

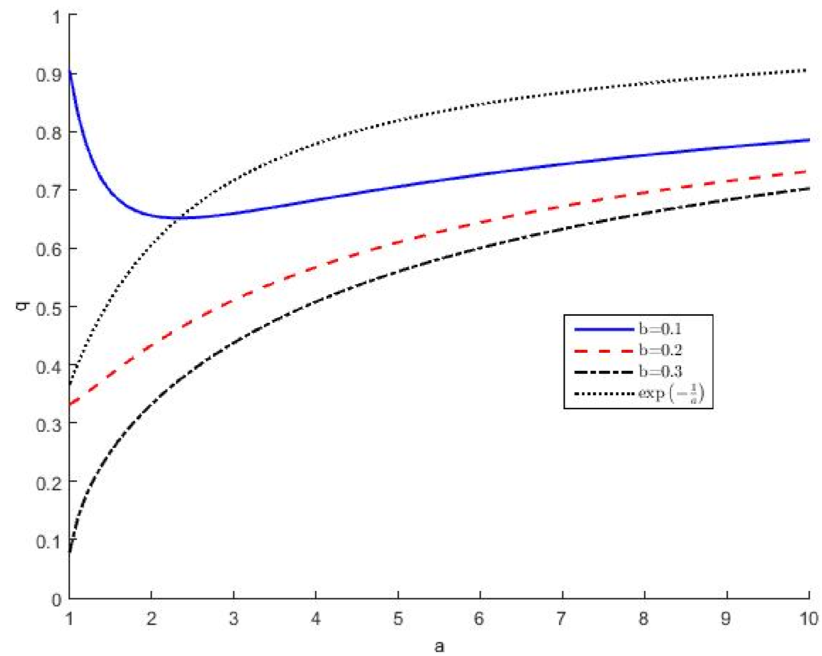


(a) Information acquisition

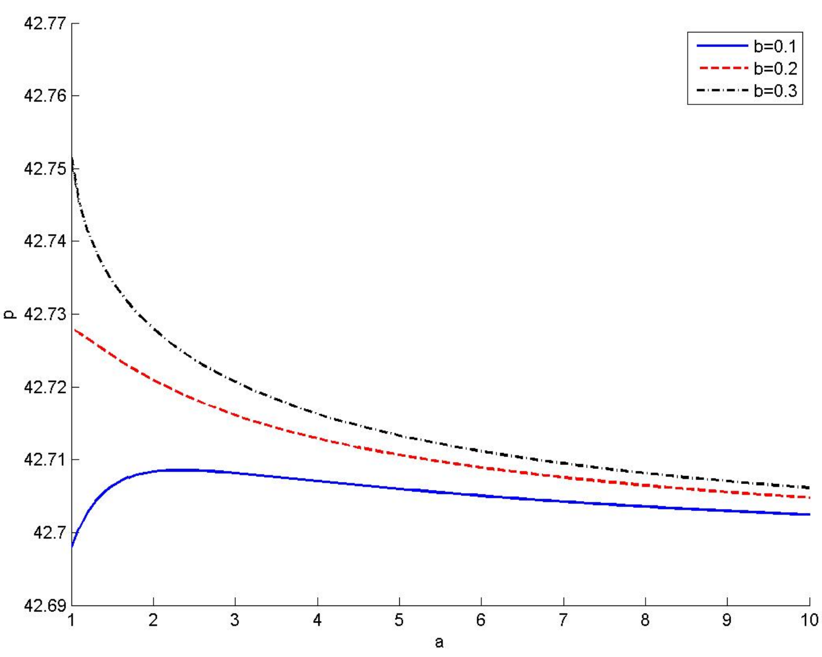


(b) Price

Figure 5: The effect of idiosyncratic component precision



(a) Information acquisition



(b) Price

Figure 6: The effect of *a*

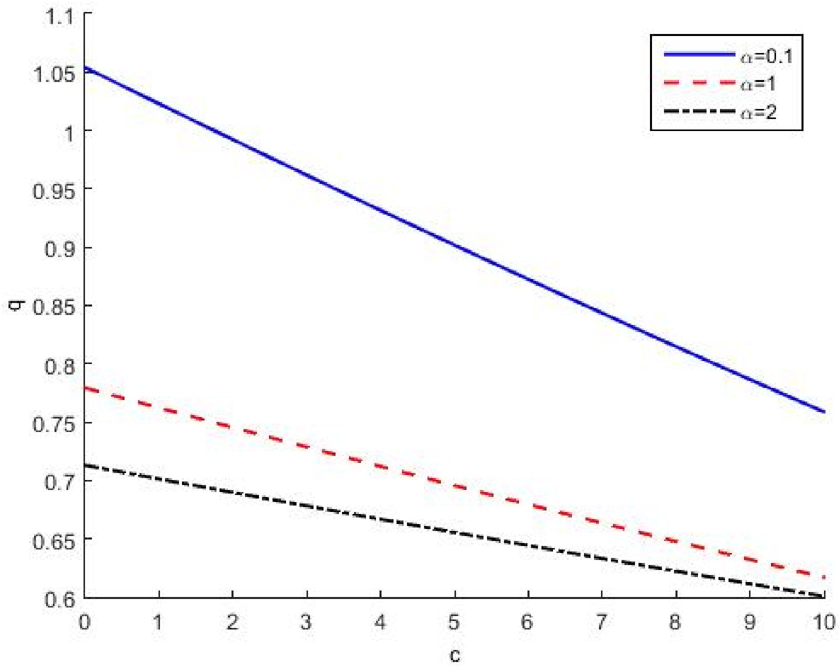
below exp $\left(-\frac{1}{a}\right)$, then the firm has greater incentives to acquire private information with larger a. Therefore, when b is very small, the increases in a at first decreases $\eta(q)$ and then increases it; with our parameter setting, this will at first increase the equilibrium price and then decreases the optimal price. When b is very large, the monopolist acquires very little information. In this case, an increase of the power a increases the equilibrium information acquisition and therefore decreases the equilibrium price.

Figures 7a and 7b show the equilibrium information acquisition and price as functions of unit production cost c when $\alpha = \beta = 1$ and information acquisition cost function is $C(q) = q^2/10$. From Equation (11), we see that given the price p, an increase in the unit production cost monotonically decreases the marginal benefit of information acquisition. Therefore, larger unit production cost monotonically decreases the equilibrium information acquisition. Figure 7a confirms this conjecture. Therefore, with a higher unit production cost, the monopolist in equilibrium has a less precise estimate of individual's ideal products.

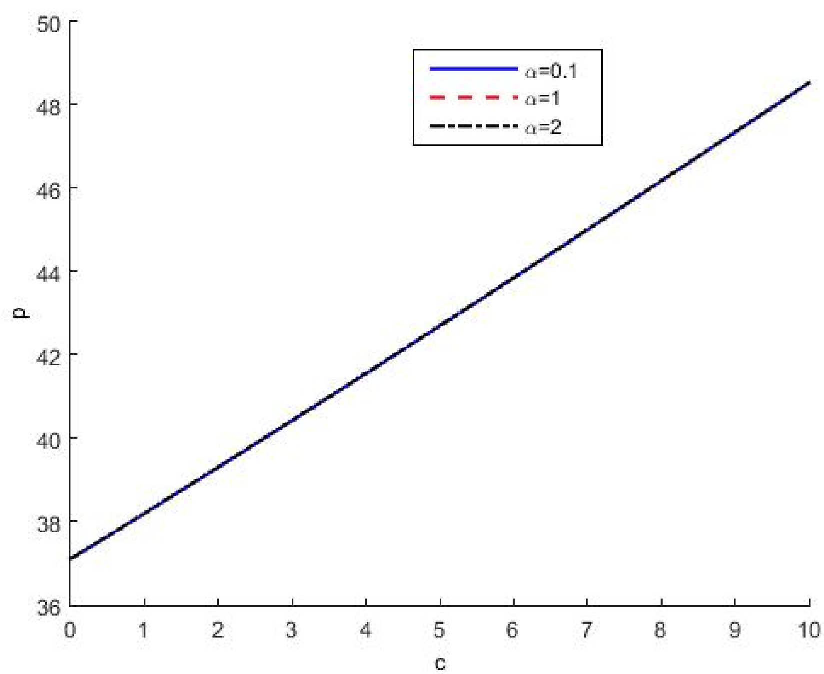
A change of unit production cost has two effects on price: firstly, with our parameter setting, decreases in $\eta(q)$ increase the monopolist's equilibrium price; secondly, as shown above increases in c imply that the firm incurs less cost by increasing the price and decreasing the demand. Therefore, higher unit cost increases the equilibrium price, this is consistent with the results of Figure 7b.

We now turn to our third batch of numerical simulations, in which we study the effect of a mean-preserving spread of consumers' valuation distribution. The reason why this may be interesting is that in general a monopolist's profit, at the monopoly price, is decreasing in the spread of consumers' valuation for the good in question. To see this, suppose, first, that all consumers have virtually the same valuation, $v \approx 50$, and that they all have product variety $\theta = 0$ as their ideal, and furthermore the unit production is 0. If the monopolist would know this, he could extract almost all consumer surplus by setting the price p just below 50 and make profits close to 50. Next, suppose instead that consumer valuations are uniformly distributed between zero and one hundred, and that again all consumers' ideal is the variety $\theta = 0$. A monopolist would know this and could only extract half the consumer surplus. He would again optimally set p close to 50, but now only half the consumers would buy, so his profits would be only 25. Hence, one may ask if this qualitative relationship holds also in our more general model. If it does, then the monopolist might invest less in information acquisition.

To find this out, we now assume that $\alpha = \beta = 1$ and $C(q) = q^2/10$, and that consumers' valuation v is log-normally distributed with mean $\bar{v}$ and variance σ^2. For our purpose, for each $\bar{v}$, we investigate how the change of σ, when keeping $\bar{v}$ constant, would affect the equilibrium information acquisition and price. Figure 8a shows that the qualitative relationship in the above very parsimonious model still holds in our more general settings: more dispersion in consumers' valuation decreases monopolist's incentive in private information acquisition.

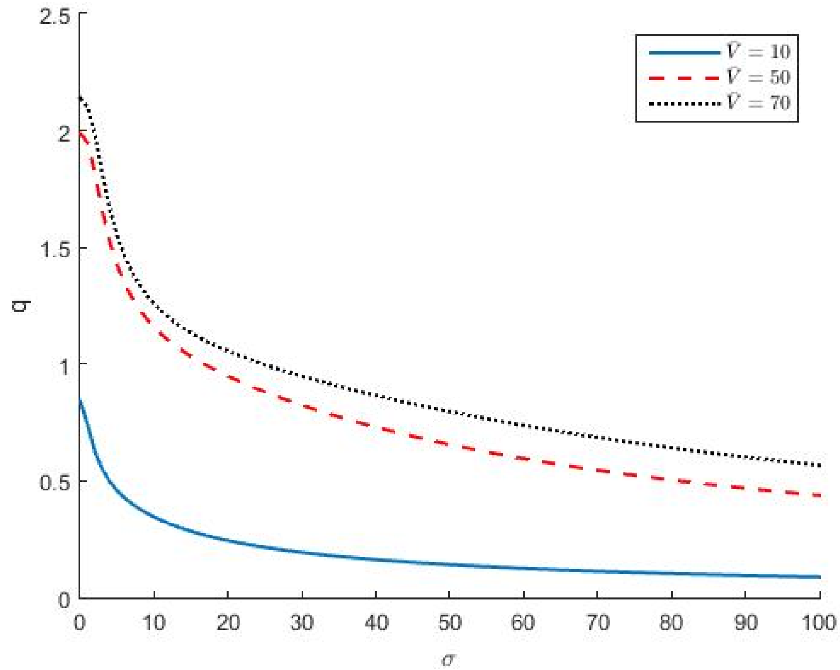


(a) Information acquisition

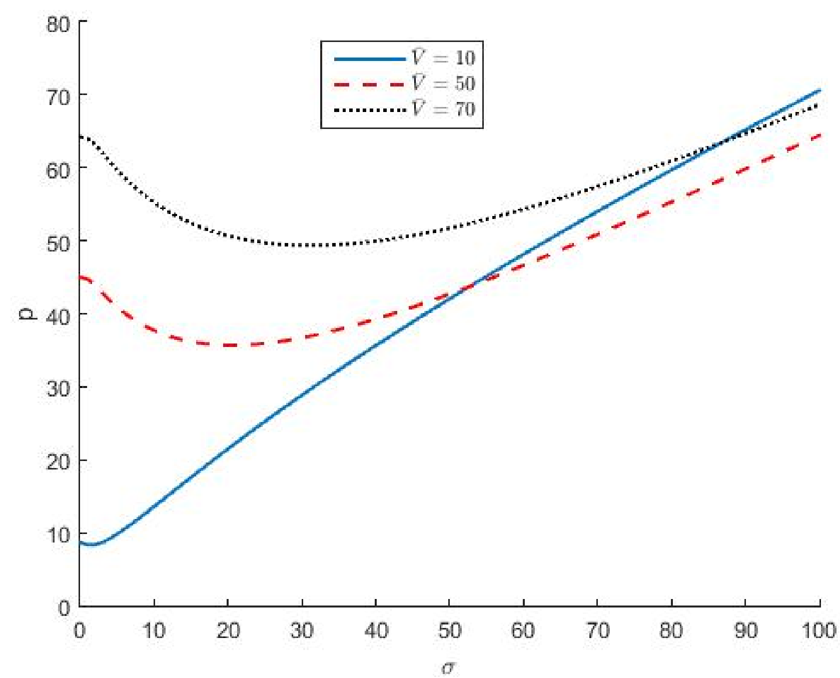


(b) Price

Figure 7: The effect of unit production cost



(a) Information acquisition



(b) Price

Figure 8: The effect of value dispersion

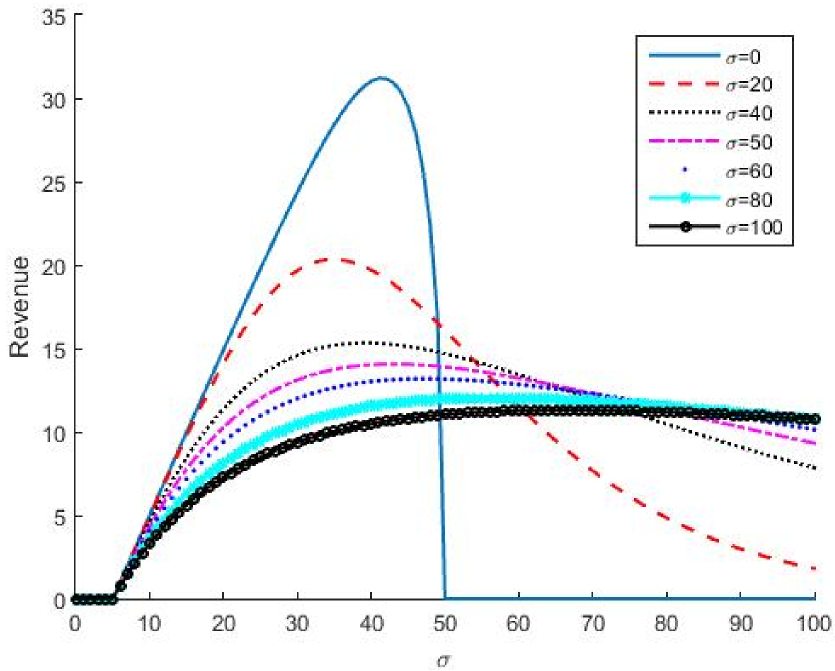
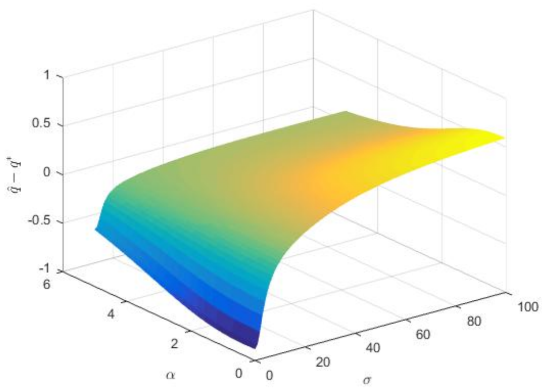


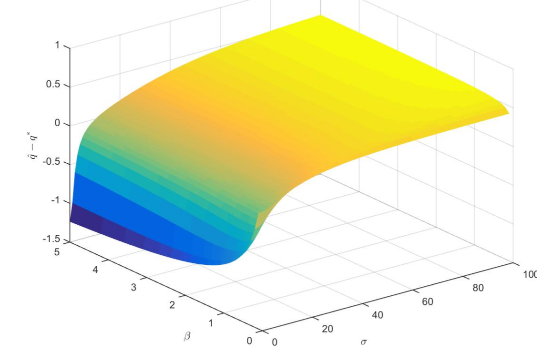
Figure 9: Revenue and price given the signal precision

Figure 8b shows the non-monotonic effects of more dispersion in valuation distribution on equilibrium price. This figure integrates two effects of more dispersion on the equilibrium price: the first one is that more dispersion decreases the equilibrium information acquisition, and therefore decreases the posterior estimate precision of individual's ideal products; the second one is that by changing the consumer's valuation distribution, the monopolist needs to adjust the price. The first effect on the price may be non-monotonic, as we have said in Remark 1, depending on the valuation distribution. However, the second effect may dominate the first one for the dispersion effects on price. Figure 9 shows that, given $\bar{v} = 50$, $\eta(q) = 1$ and $c = 5$, how optimal price is adjusted as the standard deviation of consumers' valuation distribution changes. We can see that, consistent with Figure 8b, the monopolist decreases the price when σ is between 0 and around 20 and then increases the price when σ is above 20.

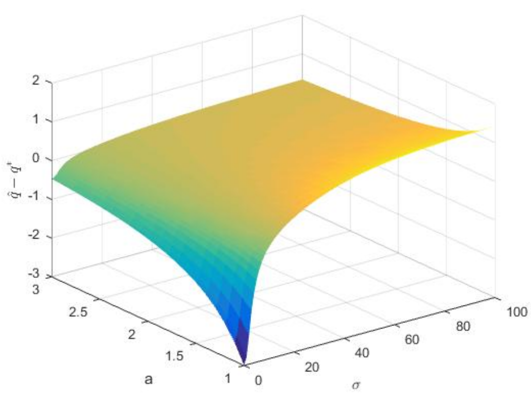
To see the effects of σ on price shown in Figure 9, we can interpret the adjustment process in terms of Figure 2. When σ is zero, all consumers have valuation $\bar{v}$ and the optimal price is shown in Equation (9) by assuming $v = \bar{v}$ and ignoring the integral. As σ increases, the mode of consumers valuation distribution decreases (see footnote 6), the shaded area and the rectangle with solid sides in Figure 2 are more valuable for the monopolist, and the monopolist prefers a lower price. As σ increases, the mode of the valuation distribution is so low that it becomes less valuable to attract



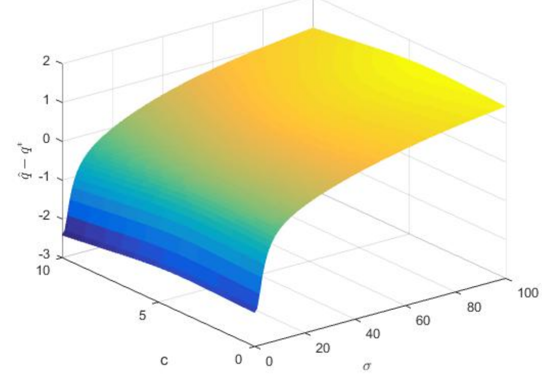
(a) (σ, α)



(b) (σ, β)



(c) (σ, a)



(d) (σ, c)

Figure 10: The effect of (α, β, a, c) on the inefficiencies

the consumers with valuation around the mode; the monopolist prefers to increasing the price to serve only the consumers with high valuation.

6.2 First best

With numerical methods, we can compare the monopolist's optimal information acquisition with the first best. Figure 10 show the difference of the signal precisions when consumers' valuation is log-normally distributed with mean $\bar{v} = 50$, and the cost function is $C(q) = bq^a$ for $b > 0$ and $a > 1$. All parameters are the same as in Section 6.1. The two figures suggest that, although both α and β have some effects on the inefficiencies of the monopoly in acquiring private information, they do *not* play a critical role in determining whether the monopolist acquires too much or too little private information. Both Figure 10 and Figure 11 show that the dispersion of consumers' valuation distribution determines the inefficiencies of the monopolist's information acquisition. When consumers are homogeneous in their valuation of the good, the monopolist invests too much in information acquisition. On the contrary, when consumers are very heterogeneous in their valuation, the monopolist has no enough incentive to acquire private information.

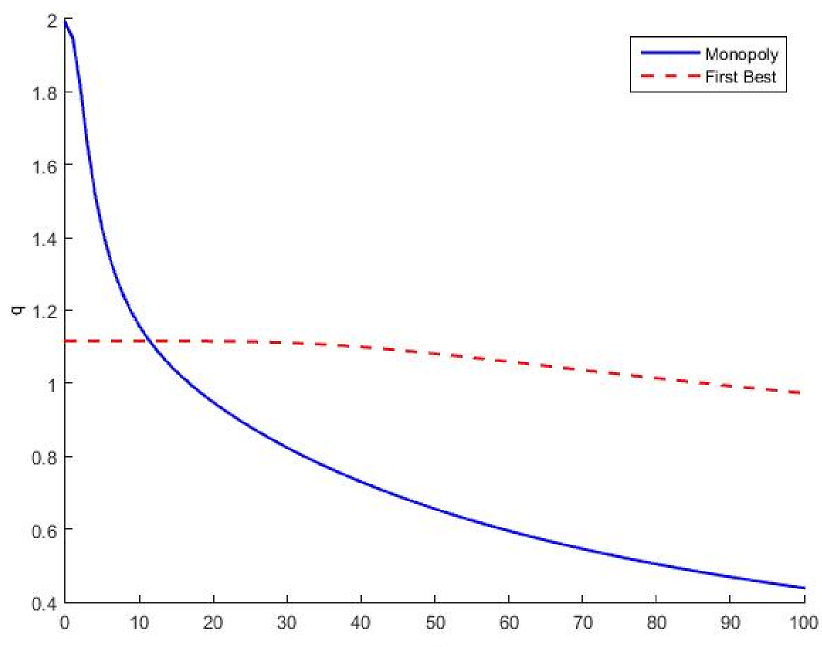


Figure 11: The effect of σ on information acquisition

To see why the valuation dispersion affects the inefficiencies of monopolist's information acquisition, we can compare the monopolist's and first-best marginal benefit of information acquisition.

Note that the marginal benefit of information acquisition in Equation (15) comes from attracting more consumers with more precise estimate of individual's ideal product. For each valuation v above the unit production cost c, the marginal benefit from attracting more consumers by investing in information acquisition is

$$\frac{1}{\eta(q)}\left(\frac{\beta}{\alpha+\beta+q}\right)^2 \int_{-\sqrt{v-c}}^{\sqrt{v-c}} t^2\phi(\eta(q)\,t)\,dt.$$

Furthermore, by a change of variables, the corresponding first-order condition Equation (11) for the monopolist's information acquisition can be reformulated as

$$\frac{p-c}{\eta(q)}\cdot\left(\frac{\beta}{\alpha+\beta+q}\right)^2 \int_{0}^{\infty} 2v^2\phi(\eta(q)\,v)\,f\left(v^2+p\right)dv.$$

This marginal benefit is the marginal benefit from attracting more consumers by investing in information acquisition multiplied by the benefit of selling out one more unit. From the expression above, we can see that for each valuation v, the marginal benefit from attracting more consumers by investing in information acquisition is approximately to be

$$\frac{2v^2\phi[\eta(q)\,v]}{\eta(q)}\cdot\left(\frac{\beta}{\alpha+\beta+q}\right)^2.$$

Figure 12 compares the monopoly's marginal benefit of attracting more consumers by investing in private information with the first-best when $\alpha=\beta=q=1$ and $c=5$. It shows that for each v, the first-best marginal benefit of attracting more consumers monotonically increases in v, and as v goes to infinity, it goes to $\beta^2/[(\alpha+\beta+q)^2\eta(q)^4]$. The private monopolist's marginal benefit of attracting more consumers is monotonically decreasing in v when v is large enough, and it tends to zero as v goes to infinity. Therefore, there exists one $\underline{v}$ such that for all $v>\underline{v}$, the first-best marginal benefit is larger than the private monopolist's, and the discrepancy between these two increases. As σ increases, $f(v)$ increases for high valuation, implying more consumers endowed with high valuation. Therefore, the discrepancy between the total marginal benefits from attracting more consumers is increasing in σ. Furthermore, in our numerical simulations, $p-c$ is generally much larger than 1; this increases monopoly's marginal benefit of information acquisition. When σ is small, the discrepancy between the total marginal benefits from attracting more consumers is *not* large enough to offset the high price in equilibrium; in this case, the monopoly has a higher marginal benefit of information acquisition. On the contrary, when σ is large enough, the discrepancy is large enough to offset the effect of high price; in this case, the monopoly has no enough incentive to acquire enough private information.

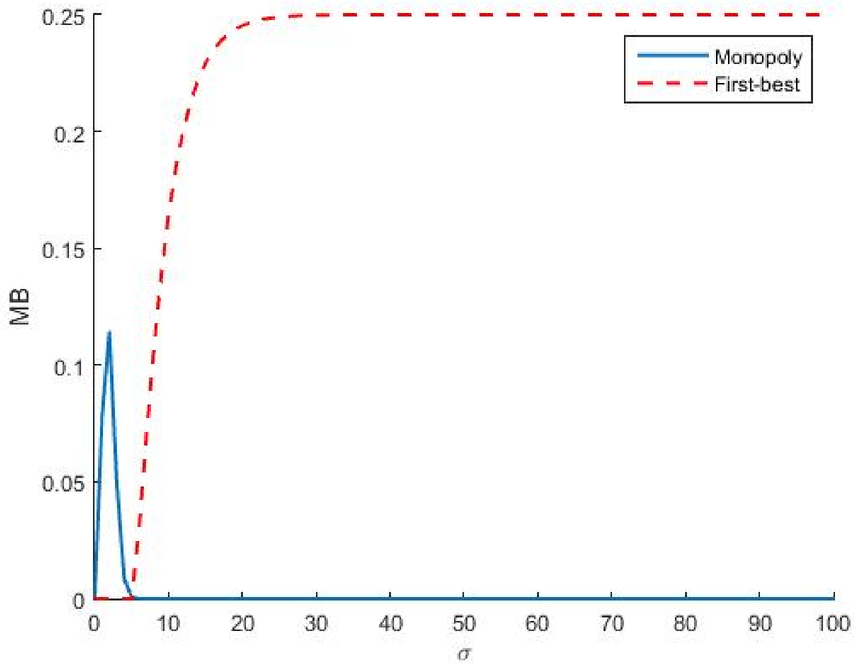


Figure 12: Marginal benefit by attracting consumers

REMARK 2. *The numerical results in this subsection are based on the log-normally distributed valuation; however, since the comparison between the marginal benefit of information acquisition to attract consumers is independent of consumers' valuation distribution, the results on the effects of the dispersion of consumers' valuation on inefficiencies in monopoly's information acquisition are robust to any kind of distribution.*

6.3 Second-best

In this subsection we numerically solve for the information acquisition and pricing when the goal function is a convex combination of the monopolist's profit and social welfare. The numerical solutions are shown in Figures 13 and 14; we assume a quadratic information acquisition cost function $C(q) = 0.1q^2$ and $c = 5$, and that v is log-normally distributed with mean $\bar{v} = 50$. We further write $\kappa = -\ln(1-\gamma)$ and assume that both the variances of shared component and idiosyncratic component are 1. We will refer to the parameter κ, which is a monotone transformation of the parameter γ, as the welfare weight in the monopolist's goal function. We will analyze the case when $\kappa \in \{10, 20, 40\}$ so that we can see how the changes in the weight affect the equilibrium pricing and information acquisition..

Figure 13 shows that the monopolist caring about both profit and welfare prefers a price between the private monopolist's optimal price and the first-best price. When

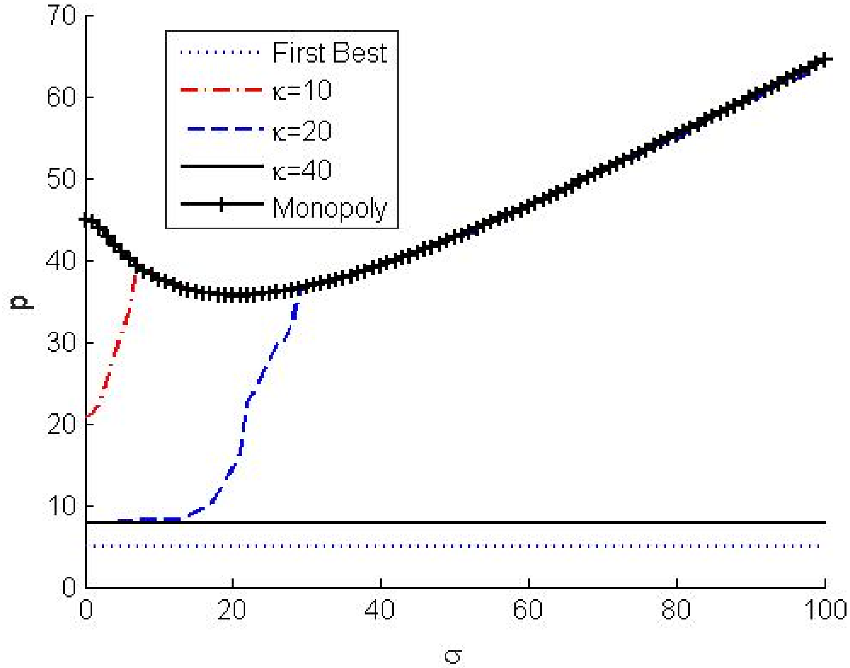


Figure 13: Price as a function of valuation distribution, for different welfare weights

γ is close to 1, the price is close to the first-best price; on the contrary, when γ is not large enough, the price is close to private monopolist's optimal price; more specifically, the monopolist's optimal price is monotonically increasing in the weight γ. Furthermore, since the profit-maximizing monopolist's optimal price may be non-monotonic in the valuation dispersion, and the marginal benefit of the monopolist caring both welfare and profit is a weighted sum of the marginal benefit to private monopoly and marginal benefit to social welfare, the optimal price for the monopolist caring both social welfare and profit and specifically the second-best price, depending on the weight γ and other parameters, may also be non-monotonic in valuation dispersion.

Figure 14 illustrates the relationship between information acquisition and valuation dispersion σ for different values of κ. When κ is large enough, the monopolist put more weight on social welfare and the information acquisition in equilibrium is more close to the first-best information acquisition; on the contrary, when κ is very small, the monopolist behaves more like the private monopolist in information acquisition. Furthermore, although both the private monopolist's and and first-best information acquisition are monotonic in consumers' valuation dispersion, the information acquisition of the monopolist caring both the profit and social welfare may be *non-monotonic* in the dispersion. Specifically, Figure 14 shows that when κ is not large enough, the monopolist's information acquisition is increasing in the dispersion

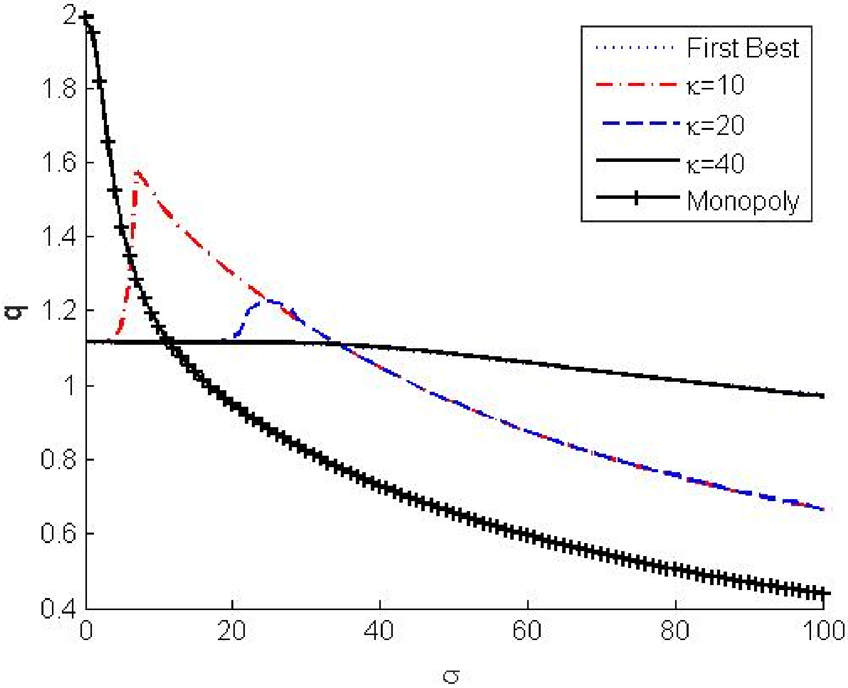


Figure 14: Information acquisition as a function of valuation distribution, for different welfare weights

when the dispersion is not large enough, and, contrarily, when the dispersion is large enough, the monopolist's information acquisition is decreasing in the dispersion.

One final comment is that, although the marginal benefit of information acquisition for the monopolist is a weighted sum of the marginal benefit to the profit and the marginal benefit to the social welfare, the optimal information acquisition of the monopolist who cares both the profits and social welfare may be higher than both the private monopolist's and first-best information acquisition. This happens when the weight is not large enough and consumers' valuation dispersion is intermediate. The intuition of this result is that, when the dispersion σ is relatively large, the marginal benefit from attracting more consumers is larger than the private monopolist; at the same time, the price of the monopolist is much higher than the first-best: In this case, the marginal benefit of information acquisition is higher than both the marginal benefit of the private monopolist and the first-best marginal benefit, resulting in an information acquisition which is higher than both private monopolist's optimal information acquisition and the first-best information acquisition. Furthermore, when the dispersion is quite small, the dispersion is not large enough to generate a high marginal benefit to the social welfare by attracting more consumers to offset the effect of the high price; in this case, the monopolist caring both the profits and social welfare acquires less information than the private monopolist and more information than the first-best. On the contrary, when the dispersion is large enough, the marginal benefit to the social welfare from attracting more consumers is large enough to offset the effect of high price; in this case, the monopolist caring both the profits and social welfare acquires more information than the private monopolist and less information than the first-best.

7 Extensions

Our model is very simple and based on heroic assumptions. We here briefly discuss a few directions in which the model may be generalized; the dimensionality of product variety, multiplicity of product variety and oligopolist competition. For while the monopolist in our model only supplies one variety and the space of varieties is one-dimensional, in practice monopolists usually provide a whole menu of product varieties where each variety has many attributes and hence is multi-dimensional. Moreover, while competition is absent from our model, in practice there is competition or at least the threat of potential entrants. While an analysis of the mentioned general cases appears quite a challenge and falls outside the scope of the present study, we here briefly sketch how the present model can be generalized.

First, in order to capture multiplicity and multi-dimensionality of varieties, the most natural generalization is arguably to let the monopolist choose a finite menu $M = \{(x_1, p_1), ..., (x_m, p_m)\}$ of product varieties and prices for these but maintain the hypothesis of unit demand. In such a setting each consumer chooses which variety, if any, to buy. Let thus each variety x_i be a vector in $X = \mathbb{R}^k$ for some positive integer k, the dimensionality of varieties, and assume a fixed cost $d(x_i) > 0$ associated with each variety, alongside its unit cost $c(x_i) > 0$ of production (where costs thus may depend on the variety). The monopolist's decision problem then is to choose a menu M from among the set $\mathcal{M} = \cup_{m \in \mathbb{N}} (X \times \mathbb{R}_+)^m$ of all finite menus, or, in other words, how many and what varieties, and what price for each variety. Given a goal function of the sort analyzed in the preceding sections, this is a challenging optimization problem. However, given such a menu $M = \{(x_1, p_1), ..., (x_m, p_m)\}$, offered by the monopolist, each consumer's choice is simple, namely, to either buy no unit or to buy one unit of the variety that gives most utility. To be more specific, let a consumer type be a pair $\tau = (\theta, v) \in X \times \mathbb{R}_+$ and let the utility for a consumer of type $\tau = (\theta, v)$ from purchasing any variety x_i at any price p_i be defined as in Section 2;

$$u_i = v - p_i - \|\theta - x_i\|^2,$$

and let

$$u_\tau(M) = \max\{u_1, ..., u_m\}.$$

Thus $u_\tau(M)$ is the utility a consumer of type $\tau = (\theta, v)$ obtains from any menu M offered by the monopolist. The consumer will buy a unit if and only if $u_\tau(M) \geq 0$. Like in Section 2, one may, for analytical tractability, treat θ and v as statistically independent random variables, each with an absolutely continuous probability distribution. If θ is multi-variate normal and the monopolists signal is additive with normally distributed noise, and if the fixed and marginal costs are the same for all varieties, then much of the preceding machinery applies, although with substantial mathematical challenge. However, for any given fixed production cost, d (assumed to be the same for all varieties), an upper bound on the number m of varieties is $m_+ = \bar{v}/d$. In order to solve (at least numerically) the monopolist's decision problem, one may then proceed by solving it first for each $m = 1, 2, .., m_+$, and thereafter choose the right number, $m^{**} \in \{0, 1, ..., m_+\}$ of varieties. Moreover, at least in the case of one-dimensional varieties ($k = 1$), the following heuristic could be explored. For $m = 1$ solve as in Sections 3 to 5 above, that is, choose variety x^{**}. For $m = 2$, place instead the two varieties at equal distance from x^{**}, and optimize over this distance. For $m = 3$, choose variety 1 to be x^{**} and place varieties 2 and 3 on each side of 1, at equal distance from x^{**}, and optimize over this distance etc.

Second, an analytically more tractable model extension that allows for multiple, but one-dimensional, varieties would be to let the monopolist select a whole "product line segment" $L = [x_o - t, x_o + t]$ as in Mussa and Rosen (1978), and require it to charge a uniform price p for all varieties in this line segment L. In this case, it is easily verified that the monopolist's optimal location of the product line segment is to center it on x^{**}, that is, to pick $x_o = qs/(\alpha + q)$. This follows from the analysis above of the special case $t = 0$ in combination with the observation that now the demand function generalizes to

$$\tilde{D}(x, p, t) = \int_p^\infty \Pr\left[x - t - \sqrt{v - p} \leq \theta \leq x + t + \sqrt{v - p}\right] f(v)\, dv$$

The optimality of the choice $x_o = qs/(\alpha + q)$ holds when the width $t \geq 0$ of the product line segment is exogenous. Suppose the width is endogenous and if the monopolist does not only have a constant unit cost $c \geq 0$ of production but also an increasing cost $e(t)$ associated with the width $t \geq 0$ of its product line, then t can be determined by a first-order condition equating the marginal cost of t, $e'(t)$, to its marginal benefit to the monopolist (see the models in Mussa and Rosen (1978) and Spence (1975)). As shown in Spence (1976a,b), this choice of t will typically be socially inefficiency when the monopolist's information about consumer preferences is fixed. An interesting avenue for further research is thus to explore this potential inefficiency in the case of endogenous information.

Third, a few words about oligopolistic competition. An extension in such a direction would be extremely interesting. The fundamental underlying question is then the effect of competition on producers' incentives to find out about consumer preferences? More precisely, suppose now that there are two profit-maximizing firms, each modelled along the lines of the profit-maximizing monopolist in Section 3, and that each firm first has to choose its signal precision, then its product variety and price. A number of alternative scenarios are possible here. Arguably, the most basic and canonical scenario is that of simultaneous-move duopoly, that is, two firms who simultaneously first choose their signal precisions, keep their signal precisions and signal realizations as their private information, and then simultaneously choose their own product variety and price. What can then be said? A conjecture for symmetric Nash equilibrium is that they will each choose the same signal precision, $q_1 = q_2 = q^*$, that they will choose their own product variety as under monopoly, that is $x_1^* = q^* s_1 / (\alpha + q^*)$ and $x_2^* = q^* s_2 / (\alpha + q^*)$, and that they will choose the same price, $p_1^* = p_2^* = p$. If so, what is then q^* and p^*, and how do these relate for the profit-maximizing monopolist's choice and to the first-best monopoly choice? And is the conjecture at all true? To explore these questions appears as a natural next step in this exploration of producers' endogenous information about consumer preferences.

8 Conclusion

This paper examines a monopolist's production variety choice, price setting and information acquisition. In our model the marginal cost of production of the good is constant, and the monopoly will choose the efficient variety. Compared with first-best, the profit-maximizing monopoly sets too high a price. However, the inefficiency in private monopoly's information acquisition depends on the parameters, especially on the consumer's valuation distribution. With log-normally distributed valuation, our numerical results show that there is a critical point of the spread such that the private monopoly acquires too much information when the spread is smaller than the critical point and the opposite is true when the spread is larger than the critical point. Compared with the second-best price setting and the information acquisition where the monopoly's profit is constrained, the profit-maximizing monopoly will generally set too high a price, and the inefficiency in the information acquisition is still determined by the consumers' valuation spread.

The results in the paper are still not as general as we would wish, and some of them rely on the numerical method. More general results on the monopoly's choice under consumers' preference uncertainty need further investigation. For instance, while we here focus the entire analysis on endogenous information about consumer preferences over product varieties, a relevant consideration is that of endogenous information about consumers' valuations of their ideal product variety. There are other important factors that may affect the value of monopoly's private information. For instance, the threat and competition from the potential entrant may also affect the monopoly's information acquisition choice (see Dimitrova and Schlee, 2003). To incorporate these factors into the model studying monopoly's information acquisition would be very interesting.

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