The Condorcet Jury Theorem with Information Acquisition *

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Abstract

We analyze a committee decision in which individuals with common preferences are uncertain which of two alternatives is better for them. Members can acquire costly information. Private signals and information choice are both continuous. As is consistent with Down's rational ignorance hypothesis, each member acquires less information in larger committees and tends to acquire zero information when the committee size goes to infinity. However, with more members, larger committees can gather more aggregate information in equilibrium. The aggregate information is infinite with the size going to infinity if and only if marginal cost at "zero information acquisition" is zero. When the marginal cost at "zero information acquisition" is positive, the probability of making an appropriate decision tends to be less than one.

Keywords: information acquisition, the Condorcet Jury Theorem, jury size, committee decision

JEL Classification: D71, D72, D81, D82

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1 Introduction

The classical Condorcet Jury Theorem argues that (i) increasing the size of committees raises the probability that an appropriate(right) decision is made, and (ii) the probability of making the appropriate decision goes to one with the size of committees going to infinity. The theorem developed out of a study by de Condorcet (1785) of decision-making process in societies when group members have private information. Recent literature on committee decision has pointed out that if information acquisition is costly, the Condorcet Jury Theorem may fail to hold. The reasoning is that each member has little incentive to acquire private information because he has a negligible probability of affecting the outcome in a large committee and thus he can free ride on the information of other members (see Gerling et al. (2005) for a survey).

Existing literature on the Condorcet Jury Theorem with information acquisition employs one of two modeling methods. In the first, members of a committee can only decide whether or not to acquire the private information; the quality of the information and the information acquisition cost is given. In these models, the proportion of members acquiring private information is non-monotone with respect to committee size, and therefore there is an optimal size that maximizes the aggregate information in the committee (see Mukhopadhaya (2003), Koriyama and Szentes (2009), Gershkov and Szentes (2009), Gerardi and Yariv (2008) and Persico (2004)). The second one makes use of binary signals and allows members to decide the quality of signals. Martinelli (2006, 2007) has shown that although each individual acquires less private information in larger committees, the probability of making an appropriate decision can be either increasing or decreasing with respect to committee size, and it does *not necessarily* go to one as the committee size tends to infinity.

We think that existing research has contributed substantially towards understanding the group decision processes with information acquisition. However, we believe that in many environments, both the signals and the quality of information choice are continuous. Arguably some results regarding the Condorcet Jury Theorem in the model with continuous signals need to be revisited.[1]

In this paper we focus on a group decision problem in which members have common preferences, but they do not know which of two alternatives is better for them. Members have no free information, but can decide how much private information

[1] Cai (2009) has studied another group-decision environment where group behavior, signals and information choice are all continuous.

they acquire. Society decides the committee size and the decision rule that defines how each member's report contributes to the final decision.

Since a committee member can observe the decision rule, each member's report will be adjusted to the decision rule. Proposition 3 and Corollary 1 show that the society cannot influence the final decision by manipulating the decision rule, and therefore the committee members' information acquisition is independent of the decision rule. This means that we can focus our attention on the parameters rather than the decision rules.

Moreover, if we see each member's report as a voting behavior on the real line, Proposition 3 shows us that linear informative voting can be an equilibrium if the decision threshold is selected carefully. This conclusion differs from Laslier and Weibull (2013), who, in a voting model with binary choice and binary signals, concludes that informative voting can be an equilibrium if and only if the individual's threshold function is the same as the society's decision rule.[2] Moreover, they show that given that the informative voting is an equilibrium the Condorcet Jury Theorem should be satisfied.[3] However, we can challenge the Condorcet Jury Theorem by adding the information acquisition choice in the model.

Proposition 4 shows us that members have less incentives to acquire private information in larger committees. This is consistent with Down's rational ignorance hypothesis and it is reasonable since the information is a public good in equilibrium, and therefore committee members are more likely to free ride on the information of others in larger committees. However, larger committees tend to gather more aggregate information, which is confirmed in Proposition 5. Therefore, to make the appropriate decision, the optimal choice for the society is to maximize the committee size when there is no participation cost for committee members.

Nevertheless, larger committees *may* invest more in information acquisition, and therefore the effect of the committee size on the social welfare is uncertain. Proposition 6 shows us two conclusions about the social welfare with information acquisition. The first is when the information acquisition cost is linear, the aggregate cost paid by committees and the social welfare are independent of committee size. The

[2] An individual's threshold function is the optimal decision threshold given that the individual has all information acquired by the society.

[3] Austen-Smith and Banks (1996) shows that the Condorcet Jury Theorem *might not* be satisfied in strategic voting by showing that informative voting may not be an equilibrium under some decision rules.

second conclusion is that when the cost function is nonlinear and when the society is large enough, larger committees are better for the society.

To be consistent with the Condorcet Jury Theorem, Proposition 7 shows the asymptotic probability of aggregate information acquired by committees. If $C'(0) = 0$ the limit of aggregate information tends to infinity and therefore the limit of the probability of making the appropriate decision tends to 1; if $C'(0)$ is positive the limit of aggregate information is finite and the society cannot obtain the appropriate decision with probability 1. Moreover we show that the limit of aggregate information is a continuous and monotonically decreasing function of $C'(0)$, with its limit being infinity when $C'(0)$ tends to zero. Combining Propositions 6 and 7 we see that the information acquisition is asymptotically efficient, and universal or near universal participation, given that the society is very large and there is no participation cost, is preferable for the society.

Next we relax the assumption that individuals are indifferent between the two choices prior to observations. Proposition 9 shows that the rational ignorance hypothesis still holds; and Proposition 10 shows that the aggregate information gathered by committees is non-decreasing in committee size. Furthermore, the limit of the aggregate information is a function of $C'(0)$. Proposition 10 also shows that this function is discontinuous, but it is continuous and monotonically decreasing when the marginal cost is small enough. It tends to infinity when the marginal cost at zero information acquisition tends to zero.

Taken together, our results show that the rational ignorance hypothesis is generally satisfied in committee decision with information acquisition, but nevertheless larger committees serve the society better than what the rational ignorance hypothesis indicates at first glance. Furthermore, the probability of making the appropriate decision *might not* be able to tend to 1. Its limit is 1 if and only if the marginal cost at zero information acquisition is zero.

Furthermore, even if committee members can only report 0 and 1, Proposition 11 and Proposition 12 show that the limit of the probability of the appropriate decision goes to 1 if and only if the marginal cost at zero information acquisition is zero and the limit is strictly less than 1 if and only if the marginal cost at zero information acquisition is positive, although the rational ignorance hypothesis still holds irrespective of the information cost function. This conclusion differs from Martinelli (2006). In a strategic voting model with binary signals, Martinelli (2006) shows that the limit of the appropriate decision goes to 1 if and only if both the marginal cost and the second order derivative at zero information acquisition are zero. The only difference between our model and the model in Martinelli (2006) is that in our model the signal space is $\mathbb{R}$ and the distribution is continuous in $\mathbb{R}$ while Martinelli (2006) assumes that the signal is binary. Therefore the continuous signal assumption really matters for the Condorcet Jury Theorem when there is information acquisition cost.

Following the work of Triossi (2013), we then extend our analysis into the model where committee members have heterogeneous information acquisition cost functions. We show that generally larger committee will acquire more information, but the probability of making the appropriate decision does not *necessarily* approach 1 when the size goes to infinity. The aggregate information acquisition goes to infinity if and only if the probability is positive for skill parameters whose marginal cost at zero information acquisition cost is zero.

Considering that in reality the signals are not restricted to be normally distributed, we then extend the analysis to more general continuous distributions. Suppose the conditional distribution satisfies the monotone likelihood ratio property. Propositions 14 and 15 show that in the limit the probability of the appropriate decision is 1 if and only if the marginal cost at zero information acquisition is zero, and if the marginal cost at zero information acquisition is positive, the limit will be strictly less than 1.

The paper proceeds as follows. Section 2 introduces the formal model. Section 3 derives the first-best solution in which the information acquisition choice is to maximize the social welfare. Section 4 derives the equilibrium, and Section 5 investigates the effects of committee size on social welfare and information acquisition in equilibrium. Section 6 extends our analysis into the model where individuals in the society are biased towards one alternative prior to observations. Section 7 does some extensions and shows that the conclusions are still applicable in other settings. Section 8 concludes the paper. All calculation and proofs are in the appendix.

2 The Model

There is a society consisting of $N(\in \mathbb{N})$ ex-an identical individuals. The underlying state of the world, $\omega \in \Omega$, can take one of the two values, 0 and 1, with common prior $\Pr(\omega = 1) = 1 - \Pr(\omega = 0) = \gamma \in (0,1)$. The society has to make a binary decision $d \in \{0,1\}$. There is no interest conflict among individuals. Each individual has a benefit $u(d,\omega)$ if decision d is made when the state of the world is ω. In particular,

$$u(d,\omega)=\begin{cases}0, & \text{if } d=\omega\\ -\alpha, & \text{if } d=0 \text{ and } \omega=1\\ -\beta, & \text{if } d=1 \text{ and } \omega=0\end{cases}$$

where $\alpha>0$ represents the severity of type-I error and $\beta>0$ indicates the severity of type-II error. [4]

The society randomly chooses n individuals to form a committee and determines the decision rule.[5] Each committee member needs to pay some cost to gather the private information about the underlying state of the world. As in Li (2001), Duggan and Martinelli (2001) and Li and Suen (2009) we assume the signals are continuous. The private signal of committee member i is

$$s_i=\omega+\varepsilon_i \text{ with } \varepsilon_i\sim\mathcal{N}\left(0,\frac{1}{q_i}\right)$$

when he pays the cost $C(q_i)$, where the cost function satisfies $C'\geq 0$, $C''\geq 0$. Furthermore, if $q_i=0$ the signal s_i is uninformative and the cost is $C(0)=0$. When $C(q_i)=cq_i$ with $c>0$ the cost function is *linear*; otherwise it is *nonlinear*. Moreover the signals between different members are conditionally independent, which implies $\text{cov}(\varepsilon_i,\varepsilon_j)=0$ for all $i\neq j$.

As in Ganuza and Penalva (2010) and Amir and Lazzati (2016), the information acquisition choice is for committee member i to choose from a family of joint cumulative distributions $\{F(s_i,\omega;q_i)\}$ indexed by the precision q_i. Suppose the probability density function of the distribution is $f(s_i,\omega;q_i)$.

Let

$$\mathbf{s}\triangleq(s_1,s_2,\cdots,s_n)$$

be the signal profile and

$$\mathbf{q}\triangleq(q_1,q_2,\cdots,q_n)$$

be the information acquisition profile.Then each information acquisition profile $\mathbf{q}$ induces a cumulative distribution function $F(\mathbf{s},\omega;\mathbf{q})$, with $f(\mathbf{s},\omega;\mathbf{q})$ being its probability density function. Denote the signal set of committee member i as $S_i=\mathbb{R}$, then the signal profile is a vector in $\mathbf{S}=\times_{i=1}^{n}S_i$.

[4]Here we assume that $d=0$ implies rejection and $d=1$ implies acceptance.

[5]When $n=N$, all individuals in the society need to join in the committee.

After receiving the private signal, the committee member i do some reports according to his private signal and the signal precision to the society

$$\varphi_i : S_i \times \mathbb{R}_+ \to \mathbb{R}$$

and the final decision is made according to the reports of all members. Let $r_i = \varphi_i(s_i, q_i)$ be the report value when the precision is q_i and the signal is s_i. Therefore, the payoff of each non-committee member is the benefit $u(d, \omega)$ and the payoff of each committee member i is the benefit net of the information acquisition cost:

$$u(d, \omega) - C(q_i).$$

The timeline is as follows:

1. The society selects the committee of size n and chooses the decision rule $\psi : \mathbb{R}^n \longrightarrow \{0, 1\}$ such that the final decision is $d = \psi(r_1, r_2, \cdots, r_n)$;

2. Knowing the committee size n and the decision rule $\psi(\cdot)$, the committee member i chooses a signal precision q_i and receives her private signal s_i;

3. All member report $r_i = \varphi_i(s_i, q_i)$. The final decision is made according to the decision rule ψ and the report profile $\boldsymbol{\varphi}(\mathbf{s}, \mathbf{q}) = \mathbf{r}$, and the payoff is realized.

We want to analyze the effects of the decision rule ψ and the member size n on committee member's behavior in equilibrium. Therefore we will try to solve the equilibrium of the game composed by n individuals in the committee. Formally the game we are studying is

$$\Gamma_\psi = \langle \mathcal{I}, \boldsymbol{\Sigma}, \mathcal{V} \rangle$$

where $\mathcal{I} = \{1, 2, \cdots, n\}$ is the set of players, $\boldsymbol{\Sigma} = \times_{i=1}^n \Sigma_i$ is the nonempty set of pure-strategy profile with $\Sigma_i \subset \mathbb{R}_+ \times \mathbb{R}$ being each player's pure strategy set and $\mathcal{V} : \Sigma \longrightarrow \mathbb{R}^n$ is the combined payoff function, where $v_i(\sigma) \in \mathbb{R}$ is i's payoff under pure-strategy profile σ. A pure strategy for player i in Γ_ψ consists of a pair (q_i, φ_i), where $q_i \in \mathbb{R}_+$ and $\varphi_i : S_i \times \mathbb{R}_+ \to \mathbb{R}$ is a Borel measurable function from the signal set and information acquisition set into reports. The payoff for i is

$$v_i(\sigma) = u(d, \omega) - C(q_i)$$

where $d = \psi(\boldsymbol{\varphi}(\mathbf{s}, \mathbf{q}))$.

Given the strategy profile σ, the expected payoff for player i is

$$\mathbb{E}[v_i(\sigma)] = \int_{\mathbf{S}\times\Omega} v_i(\sigma) dF(\mathbf{s}, \omega; \mathbf{q}) = \int_{\mathbf{S}\times\Omega} u(\psi(\boldsymbol{\varphi}(\mathbf{s}, \mathbf{q})), \omega) dF(\mathbf{s}, \omega; \mathbf{q}) - C(q_i)$$

Furthermore the social welfare is measured by the average payoff per capita:

$$W \triangleq u(d, \omega) - \frac{\sum_{i=1}^{n} C(q_i)}{N} \tag{1}$$

From the expression we can see that the society can share the benefits from the information acquired by committee members since more information is beneficial for appropriate choice and therefore increases each individual's benefit $u(d, \omega)$.

3 The First-best Solution

As a benchmark, we derive the first-best solution when there is no information asymmetries, and the society chooses the decision d and the information acquisition profile $\mathbf{q}$ to maximize the ex-ante social welfare. Since there is no information asymmetries, the decision rule is a function of the signal profile $\mathbf{s}$ and the information acquisition profile $\mathbf{q}$; so the first-best decision is $d = \kappa(\mathbf{s}, \mathbf{q})$, where the decision rule $\kappa : \mathbf{S} \times \mathbb{R}_+ \to \mathbb{R}$ is a Borel measurable function. Given the distribution $F(\mathbf{s}, \omega; \mathbf{q})$ and its probability density $f(\mathbf{s}, \omega; \mathbf{q})$, the unconditional probability density of the signal profile $\mathbf{s}$ is

$$f(\mathbf{s}; \mathbf{q}) = (1-\gamma) \prod_{i=1}^{n} f(s_i | \omega = 0; q_i) + \gamma \prod_{i=1}^{n} f(s_i | \omega = 1; q_i)$$

The problem for the society is

$$\begin{aligned} \max_{(d, q_1, q_2, \cdots, q_n) \in \{0,1\} \times \mathbb{R}_+^n} \quad & \mathbb{E}[W] \\ \text{s.t.} \quad & s_i = \omega + \varepsilon_i \text{ with } \varepsilon_i \sim \mathcal{N}\left(0, \frac{1}{q_i}\right), \forall i \end{aligned}$$

where the ex-ante social welfare is

$$\begin{aligned} \mathbb{E}[W] = & \Pr(\omega = 1)\mathbb{E}[u(d, \omega) | \omega = 1] + \Pr(\omega = 0)\mathbb{E}[u(d, \omega) | \omega = 0] - \frac{\sum_{i=1}^{n} C(q_i)}{N} \\ = & \gamma \int_{\mathbf{S}} u(\kappa(\mathbf{s}, \mathbf{q}), 1) \prod_{i=1}^{n} f(s_i | \omega = 1; q_i) d\mathbf{s} \\ & + (1-\gamma) \int_{\mathbf{S}} u(\kappa(\mathbf{s}, \mathbf{q}), 0) \prod_{i=1}^{n} f(s_i | \omega = 0; q_i) d\mathbf{s} - \frac{\sum_{i=1}^{n} C(q_i)}{N} \end{aligned}$$

Note that different from the model in Section 2, in the first best solution the final decision κ is a function of the signal profile $\mathbf{s}$ and the information acquisition profile $\mathbf{q}$ since there is no information asymmetries.

Since the signal profile is determined by the precision profile $\mathbf{q}$, the above optimization problem can be transformed into a two-stage stochastic program:

$$\begin{aligned}
\max_{\mathbf{q}\in\mathbb{R}^n_+} \quad & \left\{-\frac{\sum_{i=1}^n C(q_i)}{N} + \int_{\mathbf{S}\times\Omega} \max_{d\in\{0,1\}} \mathbb{E}[u(d,\omega)|\mathbf{s},\mathbf{q}]dF(\mathbf{s},\omega;\mathbf{q})\right\} \\
\text{s.t.} \quad & s_i = \omega + \varepsilon_i \text{ with } \varepsilon_i \sim \mathcal{N}(0, 1/q_i) \\
& \kappa : \mathbf{S}\times\mathbb{R}^n_+ \longrightarrow \{0,1\} \text{ is Borel measurable}
\end{aligned}$$

Backward induction implies that we can at first solve out the optimal decision rule in the second stage and then solve out the optimal information acquisition profile in the first stage.

In the second stage the society needs to choose the optimal decision rule given the signal profile $\mathbf{s}$ and the information acquisition profile $\mathbf{q}$:

$$\max_{d\in\{0,1\}} \mathbb{E}[u(d,\omega)|\mathbf{s},\mathbf{q}] \tag{2}$$

Given the signal profile and the information acquisition profile, we can see that the posterior estimate of the underlying state is

$$\Pr[\omega=0|\mathbf{s},\mathbf{q}] = \frac{(1-\gamma)\prod_{i=1}^n f(s_i|\omega=0;q_i)}{f(\mathbf{s};\mathbf{q})}$$

and

$$\Pr[\omega=1|\mathbf{s},\mathbf{q}] = \frac{\gamma\prod_{i=1}^n f(s_i|\omega=1;q_i)}{f(\mathbf{s};\mathbf{q})}$$

The expected payoff from the decision $d=\kappa(\mathbf{s},\mathbf{q})=0$ is

$$\Pr[\omega=0|\mathbf{s},\mathbf{q}]u(0,0)+\Pr[\omega=1|\mathbf{s},\mathbf{q}]u(0,1) = -\alpha\Pr[\omega=1|\mathbf{s},\mathbf{q}]$$

and the expected payoff from the decision $d=\kappa(\mathbf{s},\mathbf{q})=1$ is

$$\Pr[\omega=0|\mathbf{s},\mathbf{q}]u(1,0)+\Pr[\omega=1|\mathbf{s},\mathbf{q}]u(1,1) = -\beta\Pr[\omega=0|\mathbf{s},\mathbf{q}]$$

So it is optimal to choose $d=1$ if and only if [6]

$$-\beta\Pr[\omega=0|\mathbf{s},\mathbf{q}] \geq -\alpha\Pr[\omega=1|\mathbf{s},\mathbf{q}] \Longleftrightarrow \frac{\Pr[\omega=1|\mathbf{s},\mathbf{q}]}{\Pr[\omega=0|\mathbf{s},\mathbf{q}]} \geq \frac{\beta}{\alpha}$$

[6] We assume that when the two choices are indifferent, the society prefers $d=1$.

Or equivalently it is optimal to choose $d = 1$ if and only if

$$\frac{\prod_{i=1}^{n} f(s_i|\omega = 1; q_i)}{\prod_{i=1}^{n} f(s_i|\omega = 0; q_i)} \geq \frac{\beta(1-\gamma)}{\alpha\gamma}$$

Given that $f(s_i|\omega = 1; q_i)$ and $f(s_i|\omega = 0; q_i)$ are probability densities of normal distributions, with mean 0 and 1, respectively. The above expression suggests an optimal decision rule.

PROPOSITION 1. *A first-best decision rule for the society is*

$$\kappa(\mathbf{s}, \mathbf{q}) = \begin{cases} 0; & \textit{if } \frac{\sum_{i=1}^{n} q_i s_i}{Q} < s^* \\ 1; & \textit{if } \frac{\sum_{i=1}^{n} q_i s_i}{Q} \geq s^* \end{cases} \tag{3}$$

where $Q = \sum_{i=1}^{n} q_i$ *is the committee's aggregate information and*

$$s^* = \frac{1}{2} + \frac{\ln \Lambda}{Q} \tag{4}$$

with

$$\Lambda = \frac{\beta(1-\gamma)}{\alpha\gamma} \tag{5}$$

From Proposition 1 we know that the decision is determined by the threshold s^* and the weighted sum of signals $\sum_{i=1}^{n} q_i s_i / Q$. Intuitively, the larger the weighted sum of signals $\sum_{i=1}^{n} q_i s_i / Q$ is, the more likely that the underlying state is $\omega = 1$. Therefore the decision rule shows that when the weighted sum of signals is large enough the best choice for the society is to choose $d = 1$.

We also note that the decision rule in Proposition 1 is applicable when some or all signals are uninformative. For example if there is a set $\mathcal{M} \subsetneq \mathcal{I}$ such that for all $i \in \mathcal{M}$, $q_i = 0$, then the decision rule implies that the society prefers $d = 1$ if and only if

$$\frac{\sum_{i \in \mathcal{I} \setminus \mathcal{M}} q_i s_i}{\sum_{i \in \mathcal{I} \setminus \mathcal{M}} q_i} \geq \frac{1}{2} + \frac{\ln \Lambda}{\sum_{i \in \mathcal{I} \setminus \mathcal{M}} q_i}$$

This is equivalent to making the decision according to the $n - |\mathcal{M}|$ informative signals. Furthermore note that the decision rule in Equation (3) is equivalent to

$$\kappa(\mathbf{s}, \mathbf{q}) = \begin{cases} 0; & \text{if } \sum_{i=1}^{n} q_i s_i - \frac{Q}{2} < \ln \Lambda \\ 1; & \text{if } \sum_{i=1}^{n} q_i s_i - \frac{Q}{2} \geq \ln \Lambda \end{cases}$$

So when all signals are uninformative, i.e., $\mathbf{q} = \mathbf{0}$; we have $\sum_{i=1}^{n} q_i s_i - \frac{Q}{2} = 0$. Therefore the final decision is determined by the parameter Λ; this parameter is determined by the common prior and severity of the two errors.

The parameter Λ defined by Equation (5) is the cost of type-II error relative to type-I error. Li (2001) and Laslier and Weibull (2013), in committee decision models with different information structures, show that the cost in Equation (5) is critical in determining each committee's decision behavior.

In the special case $\Lambda = 1$, the threshold is $s^* = 1/2$, independent of the precision Q. Given the common prior committee members are indifferent between the two choices.

When $\Lambda > 1$, we have $\beta(1-\gamma) > \alpha\gamma$; given the common prior committee members prefer $d = 0$. In this case $s^* > 1/2$ and larger precision Q decreases the threshold. On the contrary, when $\Lambda < 1$ we have $\beta(1-\gamma) < \alpha\gamma$; given the common prior committee members prefer $d = 1$. In this case $s^* < 1/2$ and larger precision Q increases the threshold. Furthermore in both cases the threshold s^* tends to $1/2$ when the precision Q goes to infinity.

For convenience we call the model with $\Lambda = 1$ *a priori balance* and contrarily the model with $\Lambda \neq 1$ is called *a priori imbalance*.[7]

Given the optimal decision rule, the expected aggregate benefit is

$$\int_{\mathbf{S}\times\Omega} \max_{d\in\{0,1\}} \mathbb{E}[u(d,\omega)|\mathbf{s},\mathbf{q}]dF(\mathbf{s},\omega;\mathbf{q}) \tag{6}$$

$$= \int_{\sum q_i s_i/Q<s^*} -\alpha \Pr[\omega = 1|\mathbf{s},\mathbf{q}]f(\mathbf{s};\mathbf{q})d\mathbf{s} + \int_{\sum q_i s_i/Q\geq s^*} -\beta \Pr[\omega = 0|\mathbf{s},\mathbf{q}]f(\mathbf{s};\mathbf{q})d\mathbf{s} \tag{7}$$

$$= \int_{\sum q_i s_i/Q<s^*} -\gamma\alpha \prod_{i=1}^{n} f(s_i|\omega = 1;q_i)d\mathbf{s} + \int_{\sum q_i s_i/Q\geq s^*} -(1-\gamma)\beta \prod_{i=1}^{n} f(s_i|\omega = 0;q_i)d\mathbf{s} \tag{8}$$

$$= -\alpha \Pr[\omega = 1] \Pr\left[\frac{\sum_{i=1}^{n} q_i s_i}{Q} < s^*|\omega = 1\right] - \beta \Pr[\omega = 0] \Pr\left[\frac{\sum_{i=1}^{n} q_i s_i}{Q} \geq s^*|\omega = 0\right] \tag{9}$$

$$= -\alpha\gamma G_1(s^*) - \beta(1-\gamma)(1 - G_0(s^*)) \tag{10}$$

where $G_1(\cdot)$ and $G_0(\cdot)$ are conditional distributions of $\sum q_i s_i/Q$, when $\omega = 1$ and $\omega = 0$, respectively. Denote the densities by $f_1(\cdot)$ and $f_0(\cdot)$, respectively. Then we

[7] Here *a priori* suggests being "before" or "without" observation.

can see that given s^* there is

$$\alpha\gamma g_1(s^*) = \beta(1-\gamma)g_0(s^*) \tag{11}$$

and this is also the first order condition for maximizing the expected aggregate benefit in Equation (10).

Given the optimal decision in Equation (3), the society then chooses the information acquisition profile to maximize the social welfare:

$$\max_{\mathbf{q}\in\mathbb{R}_+^n} \left\{ -\alpha\gamma G_1(s^*) - \beta(1-\gamma)(1-G_0(s^*)) - \frac{\sum_{i=1}^n C(q_i)}{N} \right\}$$

where s^* is given in Equations (4) and (5).

Applying the envelope theorem and Equation (11), and taking partial derivative of Equation (10) with respect to q_i, we can solve out the social marginal value of information acquisition:

$$\nu(Q) \triangleq \frac{\beta(1-\gamma)\phi(s^*\sqrt{Q})}{2\sqrt{Q}} \tag{12}$$

where $\phi(\cdot)$ is the density of standard normal distribution.

Taking derivative of $\nu(Q)$ with respect to Q we have

$$\frac{d\nu(Q)}{dQ} = \frac{\beta(1-\gamma)\phi(s^*\sqrt{Q})}{16Q^{5/2}}\left[-Q^2 - 4Q + 4(\ln\Lambda)^2\right]$$

We can see that $\text{sgn}(d\nu(Q)/dQ) = \text{sgn}(-Q^2 - 4Q + 4(\ln\Lambda)^2)$. $-Q^2 - 4Q + 4(\ln\Lambda)^2$ is negative for all $Q \in \mathbb{R}_+$when $\Lambda = 1$, and it is monotonically decreasing in $Q \in [0, +\infty)$, and is positive at $Q = 0$ when $\Lambda \neq 1$; furthermore, it is intuitive that the unique positive real root of the equation $d\nu(Q)/dQ = 0$ given $\Lambda \neq 1$ is

$$\widetilde{Q} = 2\sqrt{1+(\ln\Lambda)^2} - 2 > 0 \tag{13}$$

Therefore when $Q > \widetilde{Q}$ the marginal value of information in the *a priori* balance model is decreasing in Q, and when $Q < \widetilde{Q}$ the marginal value of information in the imbalance model is increasing.

The social marginal value *might* be non-monotonic. We can show that $\nu(Q)$ is monotonically decreasing if and only if $\Lambda = 1$.

LEMMA 1. *(i) If $\Lambda = 1$, $\nu(Q)$ is monotonically decreasing in Q with*

$$\lim_{Q\to 0} \nu(Q) = +\infty$$

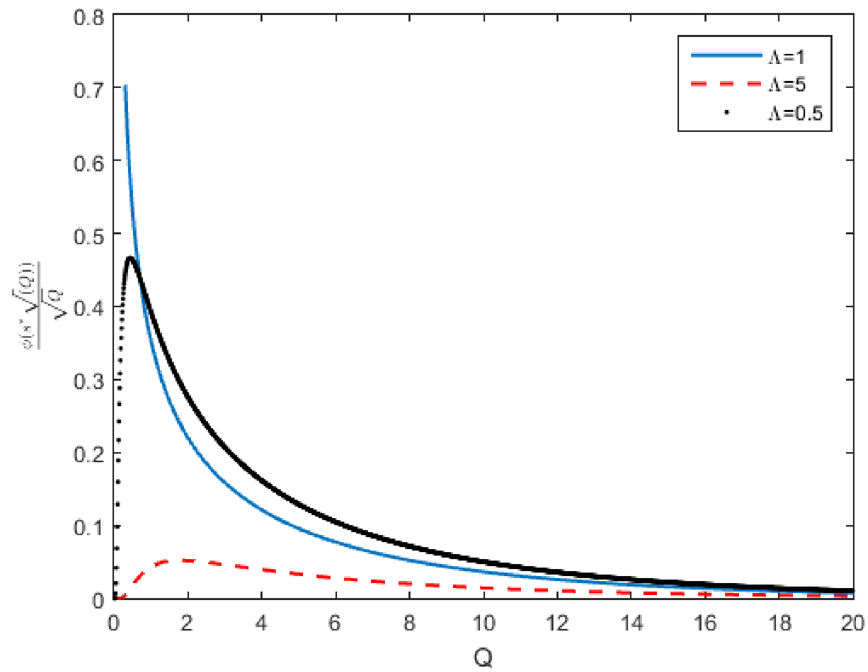


Figure 1: Marginal Value

and

$$\lim_{Q \to +\infty} \nu(Q) = 0$$

(ii) If $\Lambda \neq 1$, *then*

$$\frac{d\nu(Q)}{dQ} \lesseqqgtr 0 \text{ if and only if } Q \gtreqqless \widetilde{Q}$$

and

$$\lim_{Q \to 0} \nu(Q) = \lim_{Q \to +\infty} \nu(Q) = 0$$

When $\Lambda = 1$, the social welfare is a concave function of q_i; in this case, there will be a unique first-order information acquisition. When $\Lambda \neq 1$, the function $\nu(Q)$ is plotted in Figure 1; it is firstly increasing from 0 to its peak and then decreasing. This implies that value of the information is non-concave, it is very similar to the classic result of Radner and Stiglitz (1984) and Chade and Schlee (2002). In a principal-agent model, Lindbeck and Weibull (2016) show that the information value for the agent is similar to Figure 1. In their model the information acquisition choice is determined by the agent ability while in our model the information acquisition is determined by the cost defined in Equation (5).

Intuitively, when $\Lambda = 1$ each individual is indifferent between the two choices and therefore some information is useful to make the final decision while when $\Lambda \neq 1$ each individual prefers one choice to the other and some information is not strong enough for the society to make the final decision depending on the private information. Therefore very little information is valuable in the balance model while it is useless in the imbalance model. For clarification, we will at first discuss the *a priori* balance case, i.e. $\Lambda = 1$ and later we will show that the results can be extended to the *a priori* imbalance case, i.e., $\Lambda \neq 1$.

Given the assumption that $\Lambda = 1$, the optimal threshold of the choice is

$$s^* = \frac{1}{2}$$

and the first-order condition gives the first-best information acquisition choice; the properties of social marginal value of information acquisition guarantees the existence and uniqueness of the first-best information acquisition.

PROPOSITION 2. *Suppose $\Lambda = 1$. Then the first-best information gathering choice $\hat{q} = q$ is uniquely determined by*

$$\frac{\beta(1-\gamma)\phi(s^*\sqrt{Q})}{2\sqrt{Q}} = \frac{C'(q).}{N} \tag{14}$$

Since the social marginal value of information acquisition is determined by the aggregate information, each committee member has the same first-best information acquisition choice. Furthermore, larger society implies each agent needs to pay less cost to benefit the same aggregate information; therefore the marginal cost of information acquisition is decreasing in the society size N, and each committee member needs to acquire more in larger societies.

4 Equilibrium

In this section we want to solve the equilibrium of the game Γ_ψ given that the decision rule is ψ. Note that the society cannot observe each individual's information acquisition choice, the decision rule based on the signal quality is not applicable anymore. However, we assume that the society follows the *average decision rule*, i.e., the society takes decision $d = 1$ if and only if the average of all reports is large enough. More precisely, we assume that given the report profile $(r_1, r_2, \cdots, r_n)$, the decision rule is

$$\psi(\mathbf{r}) = \begin{cases} 0, & \text{if } \frac{r_1+r_2+\cdots+r_n}{n} < R \\ 1, & \text{if } \frac{r_1+r_2+\cdots+r_n}{n} \geq R \end{cases} \tag{15}$$

then the formal definition of the equilibrium notion under consideration is now given.

DEFINITION 1. *A pure strategy profile $(\mathbf{q}^*, \boldsymbol{\varphi}^*) \in \boldsymbol{\Sigma}$ is a Nash equilibrium with endogenous information of Γ_ψ if, for each $i \in \mathcal{I}$,*

$$(q_i^*, \varphi_i^*) \in \arg\max_{q_i \in \mathbb{R}_+, \varphi_i \in \mathbb{R}} \left\{ \int_{\mathbf{S}\times\Omega} u(\psi(\varphi_i(s_i, q_i), \boldsymbol{\varphi}_{-i}^*(\mathbf{s}_{-i}, \mathbf{q}_{-i}^*)), \omega) dF(\mathbf{s}, \omega; q_i, \mathbf{q}_{-i}^*) - C(q_i) \right\}$$

Although the game we are studying is a one-shot game, we can still distinguish between the information acquisition stage and the report stage. Following Hauk and Hurkens (2001) and Amir and Lazzati (2016), we can firstly solve the report game assuming an exogenous profile of information acquisition choice $\mathbf{q}$, then the equilibrium information acquisition choice is given by the condition that there is no incentive for any player to unilaterally deviate from $\mathbf{q}^*$; one thing we need to notice is that the committee member i's deviation can only affect his own report since the deviation is unobservable.

Therefore given the information acquisition profile $\mathbf{q}$, the equilibrium report $\boldsymbol{\varphi}^*$ should satisfy that for all $i \in \mathcal{I}$,

$$\varphi_i^*(s_i, q_i) \in \arg\max_{\varphi_i \in \mathbb{R}} \left\{ \int_{\mathbf{S}_{-i}\times\Omega} u(\psi\left(\varphi_i(s_i, q_i), \boldsymbol{\varphi}_{-i}^*(\mathbf{s}_{-i}, \mathbf{q}_{-i})\right), \omega) dF(\mathbf{s}_{-i}, \omega | s_i; q_i, \mathbf{q}_{-i}) \right\}$$

and the equilibrium information acquisition profile should satisfy that for all $i \in \mathcal{I}$,

$$q_i^* \in \arg\max_{q_i \in \mathbb{R}_+} \left\{ \begin{array}{l} \int_{S_i} \int_{\mathbf{S}_{-i}\times\Omega} u(\psi\left(\varphi_i^*(s_i, q_i), \boldsymbol{\varphi}_{-i}^*(\mathbf{s}_{-i}, \mathbf{q}_{-i}^*)\right), \omega) dF(\mathbf{s}_{-i}, \omega | s_i; \mathbf{q}_{-i}^*) dF(s_i; q_i) \\ - C(q_i) \end{array} \right\}$$

where $F(s_i; q_i) = \int_{S_i} f(s_i; q_i) ds_i$ is the unconditional distribution function of the signal s_i, with its probability density being $f(s_i; q_i) = \Pr[\omega = 1] f(s_i | \omega = 1; q_i) + \Pr[\omega = 0] f(s_i | \omega = 0; q_i)$.

We will show that given the average decision rule in Equation (15) each member can adjust the report according to the threshold R; one classic adjustment is that the report is linear in private signals. We call the equilibrium with reports linear in private signals the *linear equilibrium*. The following proposition shows that there are infinite many such equilibria.

PROPOSITION 3. *Suppose $\Lambda = 1$. Then there are infinite many linear equilibria in the game Γ_ψ. In each equilibrium the committee member $i \in \mathcal{I}$ reports*

$$r_i^* = \varphi_i^*(s_i, q_i) = a_i s_i + b_i \tag{16}$$

where

$$a_i = \lambda \cdot q_i \text{ with } \lambda \in \mathbb{R}_{++} \tag{17}$$

and b_i satisfies

$$\sum_{i=1}^{n} b_i = nR - \frac{\lambda}{2} Q - \lambda \ln \Lambda \tag{18}$$

and acquires the private information $q^* = q > 0$, which is uniquely determined by

$$v(Q) = C'(q) \tag{19}$$

where $Q = nq$.

Note that given equilibrium report shown in Equations (16) to (18) we have

$$r_i^* | \omega \sim \mathcal{N}\left(\lambda q_i \omega + b_i, \lambda^2 q_i\right)$$

and therefore

$$\sum_{i=1}^{n} r_i^* | \omega \sim \mathcal{N}\left(\lambda Q \omega, \lambda^2 Q\right)$$

So given the information acquisition profile $\mathbf{q}$ and the report strategy in Equations (16) to (18), the expected utility is

$$\begin{aligned}
&\mathbb{E}[u(\psi(\mathbf{r}^*), \omega)] - C(q_i) \\
&= -\alpha \Pr[\omega = 1] \Pr\left[\sum_{i=1}^{n} r_i^* < nR | \omega = 1\right] - \beta \Pr[\omega = 0] \Pr\left[\sum_{i=1}^{n} r_i^* \geq nR | \omega = 0\right] - C(q_i) \\
&= -\alpha\gamma\Phi\left[(s^* - 1)\sqrt{Q}\right] - \beta(1-\gamma)\left\{1 - \Phi\left[s^*\sqrt{Q}\right]\right\} - C(q_i) \\
&= \mathbb{E}[u(\kappa(\mathbf{s}, \mathbf{q}), \omega)] - C(q_i)
\end{aligned}$$

where $\Phi(\cdot)$ is the distribution function of standard normal distribution, s^* is defined in Equations (4) and (5), and $\kappa(\mathbf{s}, \mathbf{q})$, which is defined in Equation (3), is the optimal decision rule given the society can observe all signals and the information acquisition choices. Therefore the reports in equilibrium should be that the final decision according to the decision rule ψ in Equation (15) is the same as that all signals and information acquisition choices are observed directly and the decision rule is κ in Equation (3). This is because there is no interest conflict among committee members. From Equation (16) we know that the parameter a_i, which is linear in the precision q_i, is to adjust the variance of the conditional distribution of the average report $\sum_{i=1}^{n} r_i/n$ so that it is consistent with the variance of the conditional distribution of $\sum_{i=1}^{n} q_i s_i/Q$. On the other hand, b_i is adjusted according to the threshold R and the parameter λ so that the final decision according to ψ is independent of the threshold. Because of this both of the expected utility and the equilibrium information acquisition is independent of the threshold R. Formally,

COROLLARY 1. *All average decision rules are outcome-equivalent, that is to say, the threshold R cannot affect the final decision and the information acquisition choice in any linear equilibrium.*

Therefore the society cannot benefit from manipulating the threshold of the average decision rules. Different average decision rules cannot affect the equilibrium information acquisition, this is confirmed by Equation (19).

From Equation (19) we know that the marginal value of information is a function of the aggregate information gathered by committees; this is because the decision according to the reports profile $\mathbf{r}$ and the decision rule ψ is the same as the decision from the signal profile $\mathbf{s}$,the information acquisition profile $\mathbf{q}$ and the decision rule κ. Therefore the information is fully shared and one member's information acquisition can benefit other members. Therefore the marginal value of information is the same for all committee members; since all members have the same information acquisition cost function; all members would acquire the same private information in equilibrium.Moreover, since all individuals in the society have no interest conflicts and the social welfare is measured by the utility per capita, the marginal benefit equals the social marginal value of information. Furthermore, as shown in Lemma 1

$$\lim_{q \to 0} \nu(Q) = +\infty$$

and

$$\lim_{q \to +\infty} \nu(Q) = 0$$

Therefore Equation (19) has a unique positive solution for any information acquisition cost function. Furthermore, since each committee member's information acquisition choice does not take into consideration other individuals' benefit from the information acquisition, committee members cannot acquire efficiently sufficient private information. Formally,

COROLLARY 2. *Suppose $\Lambda = 1$. Then for each committee with size $n \in \mathbb{N}$, $q^* < \hat{q}$ and $d(\hat{q}/q^*)/dN > 0$.*

The above corollary also shows that the gap between the first-best and the equilibrium information information acquisition is larger in larger societies. This is because in larger societies more individuals can benefit from the information acquisition, and therefore the first-best information acquisition is monotonically increasing in the society size. On the contrary the equilibrium information acquisition is independent of the society size since each member only takes their own benefit from information acquisition into consideration.

REMARK 1. *Suppose the society can observe each member's information acquisition choice. Now the game is two-stage. Then the equilibrium report is solved in the report game with an exogenous profile of information acquisition* $\mathbf{q}$. *And the information acquisition in equilibrium now is given by*

$$q_i^* \in \arg\max_{q_i \in \mathbb{R}_+} \left\{ \begin{array}{l} \int_{S_i} \int_{\mathbf{S}_{-i} \times \Omega} u(\psi(\hat{\varphi}_i(s_i, \mathbf{q}_{-i}^*, q_i), \boldsymbol{\varphi}_{-i}^*(\mathbf{s}_{-i}, \mathbf{q}_{-i}^*, q_i), \omega) dF(\mathbf{s}_{-i}, \omega | s_i; \mathbf{q}_{-i}^*) dF(s_i; q_i) \\ - C(q_i) \end{array} \right\}$$

Note that this condition needs to consider the strategic effects of information acquisition on other members' report behavior. However the reporting behavior equilibrium in Equations (16) to (18) shows that the equilibrium payoff is the same as the planner knows each member's signal and information acquisition in the second stage. Therefore the information acquisition choice in the game is still given by Equation (19).

5 Committee Size and Social Welfare

In this section we want to discuss the effects of committee size on the social welfare and the information acquisition choice in equilibrium. We denote by $q^*(n, N)$ each committee member's information acquisition in equilibrium when the society size is $N \in \mathbb{N}$ and there are $n \leq N$ members in the committee; $Q^*(n, N) = nq^*(n, N)$ is thus the aggregate information gathered by committees in equilibrium when the society size is N and the committee size is $n \leq N$. The limits of individual information acquisition and aggregate information acquisition are defined as

$$\lim_{n \to +\infty} \lim_{N \to +\infty} q^*(n, N) \tag{20}$$

and

$$\lim_{n \to +\infty} \lim_{N \to +\infty} Q^*(n, N) \tag{21}$$

The limits of information acquisition in Equations (20) and (21) is understood as at first the society size tends to infinity and then the committee size tends to infinity. Recall that given $n \leq N$, both $q^*(n, N)$ and $Q^*(n, N)$ are independent of N, we can define the limits of information acquisition in another way: suppose there is one non-decreasing function $g : \mathbb{N} \to \mathbb{N}$ satisfying $g(N) \leq N$ and $\lim_{N \to +\infty} g(N) = +\infty$; then the limits of information acquisition are

$$\lim_{N \to +\infty} q^*(g(N), N) \tag{22}$$

and

$$\lim_{N \to +\infty} Q^*(g(N), N) \tag{23}$$

Given the information acquisition is independent of committee size, the two definitions of limits of information acquisition are equivalent to each other. Note that both definitions are to make the committee size tend to infinity and the information acquisition is independent of N; with some abuses of notation, we denote $q^*(n) \triangleq q^*(n, N)$ and $Q^*(n) \triangleq Q^*(n, N)$ given $n \leq N$ and therefore the limits of information acquisition is denoted as

$$\lim_{n \to +\infty} q^*(n) \triangleq \lim_{n \to +\infty} \lim_{N \to +\infty} q^*(n, N) \equiv \lim_{N \to +\infty} q^*(g(N), N)$$

and

$$\lim_{n \to +\infty} Q^*(n) \triangleq \lim_{n \to +\infty} \lim_{N \to +\infty} Q^*(n, N) \equiv \lim_{N \to +\infty} Q^*(g(N), N)$$

5.1 Rational Ignorance

This subsection focuses on the effects of finite committee size. Note that the marginal benefit is monotonically decreasing in the committee size, each individual has less incentives to acquire private information in a larger committee.

PROPOSITION 4. *Suppose $\Lambda = 1$. Then in any linear equilibrium*

$$\frac{dq^*(n)}{dn} < 0 \tag{24}$$

and

$$\lim_{n \to +\infty} q^*(n) = 0 \tag{25}$$

This proposition is consistent with Down's rational ignorance hypothesis (Downs, 1957a,b): each individual has less incentives to invest in political information acquisition in larger committees, and each individual tends to acquire no private information as the size of the committee goes to infinity.

Figure 2 shows each individual's information acquisition in equilibrium when $\alpha = \beta = 6$ and $\gamma = 0.5$, and the cost function is $C(q) = q^2/10$. It validates the conclusion in Proposition 4. The reasoning behind the conclusion is very simple: with larger committee, it is more beneficial to free ride on other member's private information.

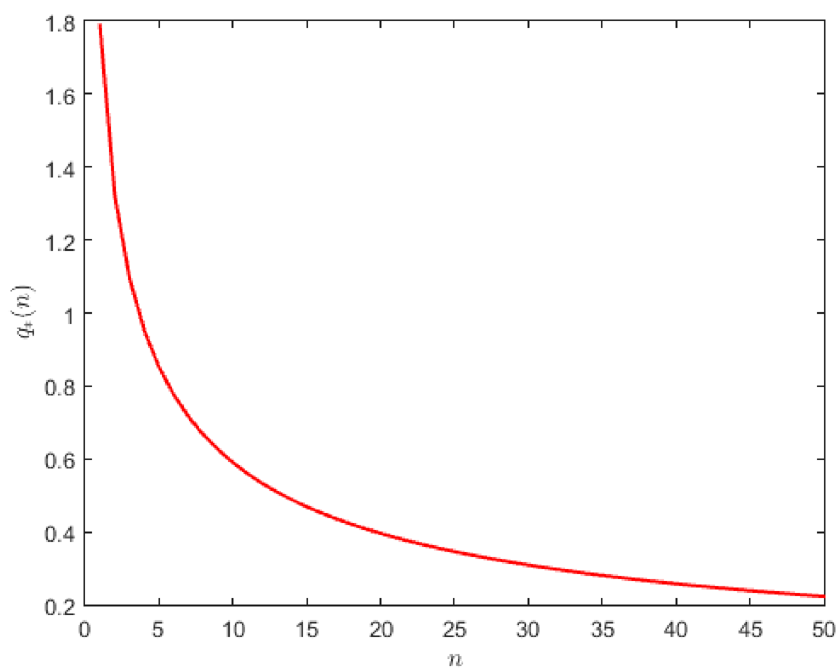


Figure 2: Committee Member's Information Acquisition in Equilibrium

Nevertheless larger committees need to provide enough aggregate information for each individual to free ride on the committee. The following proposition shows that the society is more well informed with larger committees, although each individual in the committee acquires less private information in equilibrium.

PROPOSITION 5. *Suppose $\Lambda = 1$. Then in any linear equilibrium*

$$\frac{dQ^*(n)}{dn} \geq 0$$

with equality if and only if the information acquisition cost is linear.

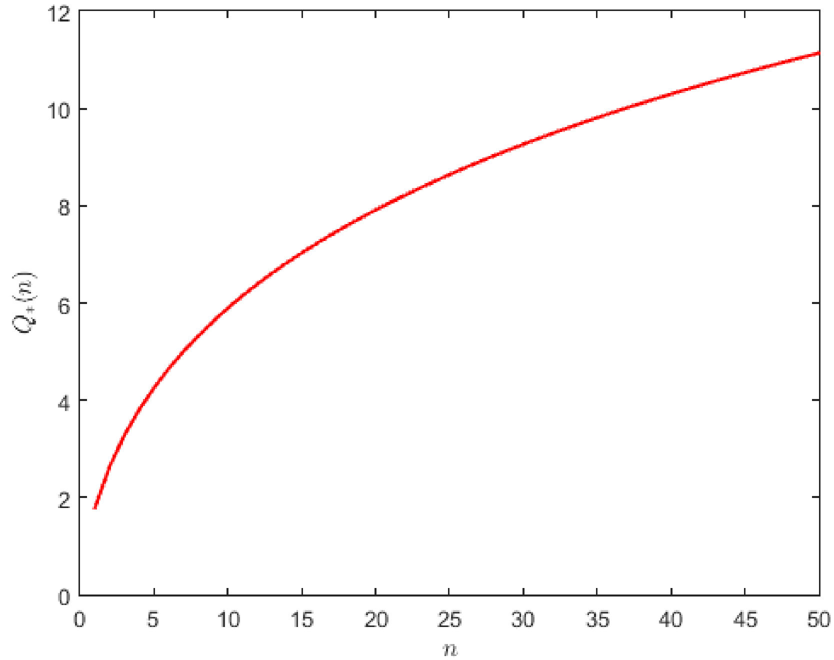


Figure 3: Aggregate Information in Equilibrium

This proposition has two conclusions: when the information acquisition cost is linear, the society cannot affect the aggregate information in the committee by manipulating the committee size; and when the information acquisition cost is nonlinear, larger committee will gather more information in equilibrium. Figure 3 shows the aggregate information gathered by the committee as its size becomes larger when $\alpha = \beta = 6$ and $\gamma = 0.5$, and the cost function is $C(q) = q^2/10$; it confirms the proposition.

Note that given the aggregate information, the probability of the right decision is

$$P(n) \triangleq \gamma\Phi\left[-(s^* - 1)\sqrt{Q^*(n)}\right] + (1-\gamma)\Phi\left[s^*\sqrt{Q^*(n)}\right] \tag{26}$$

where $\Phi(\cdot)$ is the standard normal distribution. The derivative of the probability with respect to $Q^*(n)$ is

$$\frac{dP(n)}{dQ^*(n)} = \gamma\phi\left[\left(\frac{1}{2} - \frac{\ln\Lambda}{Q^*(n)}\right)\sqrt{Q^*(n)}\right]\frac{Q^*(n) + 2\ln\Lambda}{4(Q^*(n))^{3/2}} + (1-\gamma)\phi\left[\left(\frac{1}{2} + \frac{\ln\Lambda}{Q^*(n)}\right)\sqrt{Q^*(n)}\right]\frac{Q^*(n) - 2\ln\Lambda}{4(Q^*(n))^{3/2}}$$

It is positive given $\Lambda = 1$. Therefore, more aggregate information will decrease the probability of wrong decisions.

COROLLARY 3. *Suppose $\Lambda = 1$. Then in any linear equilibrium*

$$\frac{dP(n)}{dn} \geq 0$$

with equality if and only if the information acquisition cost is linear.

When the information acquisition cost function is linear, Proposition 5 shows that the aggregate information is constant in committee size; it is intuitive that the probability of right decision is constant. Contrarily when the cost function is nonlinear it is more likely to make the appropriate decision for the society with larger committees, and this will reduce the cost of false decisions. Nevertheless, the aggregate cost borne by committee members *may be* larger in larger committees, this will affect the social welfare. However, the following proposition provides some results for the welfare effects of the committee size.

PROPOSITION 6. *Suppose $\Lambda = 1$. Then*

(i) if the information acquisition cost is linear, the social welfare is constant in the committee size n in any linear equilibrium;

(ii) if the information acquisition cost is nonlinear, there exists an N_0 such that when $N > N_0$, the social welfare is increasing in the committee size n in any linear equilibrium.

The first part of Proposition 6 is a direct conclusion of Proposition 5: with linear information acquisition cost function, the aggregate information gathered by committees is constant in committee size; therefore the probability of making the appropriate decision and the aggregate cost are both constant in committee size.

When the information acquisition cost function is nonlinear, the aggregate cost *might* be increasing in committee size since there is more aggregate information in larger committees. Therefore the social welfare *might* be decreasing in committee size. The proof of Proposition 6 shows that when the society is large enough the benefit of increasing committee size can offset the information acquisition cost from larger committees; this is because more individuals are benefiting from more information gathered by larger committees in our model and more individuals are bearing the more costs from larger committees. More specifically one sufficient condition for the second part of Proposition 6 is

$$N \geq N_0 \triangleq \frac{C\left(q^*(n)\right)}{u\left(Q^*(n)\right) - u\left(Q^*(n-1)\right)} \tag{27}$$

Note that when

$$C(q^*(n)) \leq u(Q^*(n)) - u(Q^*(n-1)) \text{ for all } n \in \mathbb{N} \tag{28}$$

we have $N_0 \leq 1$ and therefore the Inequality (27) is satisfied for all N. This implies that the social welfare is monotonically increasing in committee size for all committees.

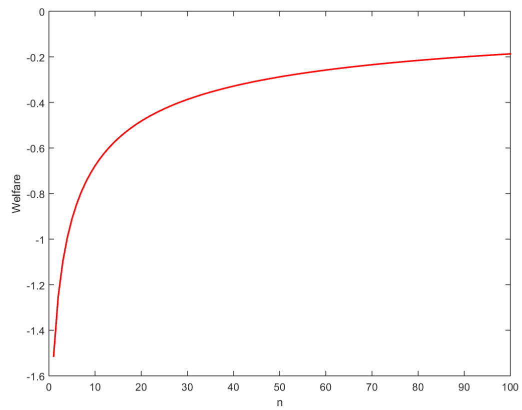
Figure 4 shows us one environment where the social welfare is increasing in the committee size. It is the solution when $N = 100$, $\alpha = \beta = 6$, $\gamma = 0.5$ and $C(q) = q^2/10$. Figure 4b shows that the aggregate cost is non-monotone in committee size: it is increasing when the committee is very small and is decreasing when the committee is large enough. However, no matter how the aggregate cost is changed, Figure 4a shows that the social welfare is monotonically increasing in the committee size; this means that the society can benefit more from more aggregate information gathered by larger committees.

The above analysis is based on the assumption that there is no participation cost, which refers to the cost paid to become one committee member. Adding the participation cost into the model will change the conclusion very much (see the discussion in Cai (2009)). For example, the following corollary shows that the society prefers small committees when there is participation cost and the information acquisition cost is linear.[8]

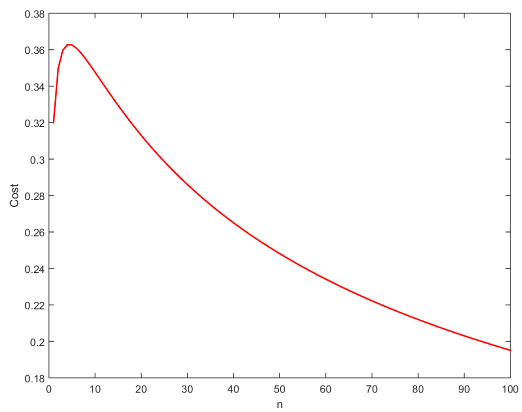
COROLLARY 4. *Suppose there is participation cost, and the information acquisition cost function is linear, the optimal committee size is* $n = 1$.

The above conclusion is consistent with Proposition 6. With linear cost function, the aggregate information gathered by committees is independent of the size; since

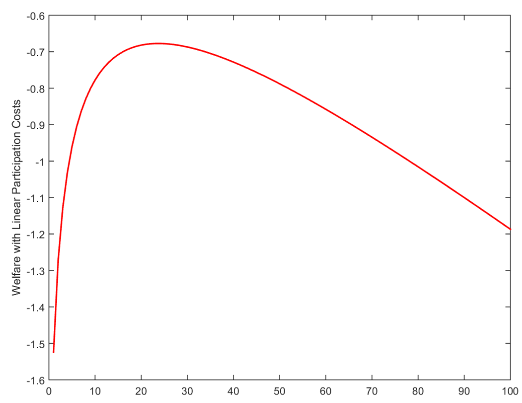
[8]For more discussion on the participation cost on the effects of Condorcet Jury Theorem and the voting behavior, see Krishna and Morgan (2012) and the references therein.



(a) Social Welfare



(b) Aggregate Cost



(c) Social Welfare with Linear Participation Costs

Figure 4: Welfare Analysis

larger committee requires more participation costs, it is very intuitive that the society prefers smaller committees.

Furthermore, when the cost function is nonlinear, the participation cost might force the society to choose an optimal committee size less than the society size. Figure 4c shows us the optimal committee size when $\alpha = \beta = 6$, $\gamma = 0.5$, $N = 100$, and the cost function is $C(q) = q^2/10$, and the participation cost is n, which means the society needs to pay the cost 1 unit for each member. The calculation result shows that the society prefers a committee with about 24 members; it is much smaller than the society size N.

5.2 The asymptotic results

In this subsection, we analyze the limiting properties of the equilibrium when committee size tends to infinity. From the equilibrium information acquisition described in Equation (18), we have the following proposition.

PROPOSITION 7. *Suppose* $\Lambda = 1$. *Then*

(i) if $C'(0) = 0$,

$$\lim_{n \to +\infty} Q^*(n) = +\infty;$$

(ii) if $C'(0) = c > 0$,

$$\lim_{n \to +\infty} Q^*(n) = \nu^{-1}(c).$$

The above proposition shows the second aspect of the Condorcet Jury Theorem when there is information acquisition choice. It verifies the significance of the information acquisition in determining the asymptotic results in committee decisions.

The first part of Proposition 7 proves the Condorcet Jury Theorem when there is information acquisition. Since the information gathered by committees in equilibrium tends to infinity with the size going to infinity, the probability of making the appropriate decision goes to one. The sufficient and necessary condition for this is that the marginal cost of information acquisition is zero when there is no information acquisition. The first panel of Table 1 shows the numerical results when the cost function is $C(q) = q^2/10$, $\alpha = \beta = 6$ and $\gamma = 0.5$; although each member acquires less information in larger committees in the second column, the third column shows that the aggregate information is monotonically increasing and the fourth column shows that the probability of the appropriate decision tends to one.

Table 1: Aggregate information with different cost functions

n	$q^*(n)$	$Q^*(n)$	$P(n)$
$C(q) = q^2/10$:			
1	1.78884	1.78884	0.74817
11	0.55920	6.15116	0.89253
101	0.13872	14.01075	0.96937
1001	0.02525	25.27835	0.99403
10001	0.00386	38.60931	0.99905
100001	0.00053	53.18525	0.99987
1000001	0.00007	68.55616	0.99998
$C(q) = q^2/100 + q/100$:			
1	5.67655	5.67655	0.88323
11	1.16980	12.86781	0.96356
101	0.18364	18.54772	0.98435
1001	0.02034	20.35985	0.98797
10001	0.00206	20.59960	0.98838
100001	0.00021	20.62438	0.98842
1000001	0.00002	20.62687	0.98842
$C(q) = q\exp(q)/100$:			
1	2.23571	2.23571	0.77265
11	0.94834	10.43169	0.94683
101	0.18136	18.31779	0.98382
1001	0.02034	20.35627	0.98796
10001	0.00206	20.59956	0.98838
100001	0.00021	20.62438	0.98842
1000001	0.00002	20.62687	0.98842

When $C'(0) \neq 0$, Proposition 7 shows that the Condorcet Jury Theorem is *not* valid any more. The intuition is very simple. If the information gathered by committees is infinite, the marginal benefit of information acquisition is zero; and therefore the marginal cost is higher than the marginal benefit, no member would acquire information. Therefore aggregate information gathered by committees must be finite. Furthermore, Proposition 7 shows that two committees with two different cost functions will tend to gather the same aggregate information as long as the marginal cost at $q = 0$ equals. The second and the third panel of Table 1 shows the calculation results when $C'(0) = 0.01$, $\alpha = \beta = 6$ and $\gamma = 0.5$. The second column shows that each member tends to acquire no private information in larger committees. But the aggregate information gathered by committees is monotonically increasing in the size. There is a finite limit when the committee size tends to be infinite, which is around 20.62687 with both cost functions. In both cases, the probability of making the appropriate decision tends to 0.98842, much less than 1.

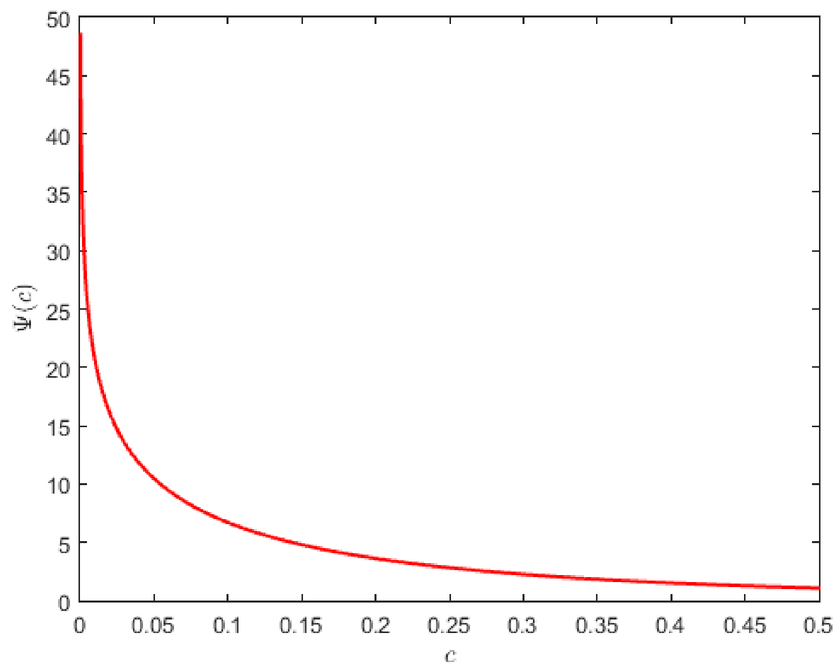


Figure 5: Asymptotic Aggregate Information

Our results differ very much from Koriyama and Szentes (2009) in which each member can only decide to acquire the information or not but not the information quality choice. They show that the Condorcet Jury Theorem is not satisfied while our model shows that when the information acquisition is continuous, the cost function plays a key role in evaluating the Condorcet Jury Theorem.

Furthermore, the conclusions in Proposition 7 are very similar to the ones in Burguet and Vives (2000): in one social learning model with information acquisition, Burguet and Vives (2000) show that the aggregate information gathered over time tends to infinity if and only if the marginal cost at zero information acquisition is zero; in our model, the aggregate information is changing in the committee size but not the time.

We have shown that for each $C'(0) = c$ the aggregate information gathered by committees with infinite size is uniquely determined, we can define the limit of aggregate information as a function of c:

$$\Psi(c) \triangleq \lim_{n \to +\infty} Q^*(n) \tag{29}$$

then Proposition 7 shows that $\Phi(c)$ is continuous and monotonically decreasing in c, this is shown in Figure 5. It shows that when c tends to zero, $\Psi(c)$ tends to infinity; and when c tends to infinity, $\Psi(c)$ tends to zero.

6 A Priori Imbalance

In this section, we want to extend the analysis of *a priori* balance model into *a priori* imbalance model, that is, $\Lambda \neq 1$. First of all, note that when $\Lambda \neq 1$, the marginal value of information is defined by $v(Q)$ in Equation (18), with s^* being defined by Equations (4) and (5). Furthermore we have shown that the value of information is non-concave in the imbalance model. However, different from Radner and Stiglitz (1984), the value of information, as seen from Lemma 1, is composed by two parts: when Q is less than $\widetilde{Q}$, the value of information is convex, and when Q is larger than $\widetilde{Q}$, the value of information is concave.

Therefore, the relationship between the marginal cost and marginal value of information acquisition has three cases: the first case is that the marginal cost is always greater than marginal value, the second case is that the marginal cost and the marginal value have two intersections and the two intersections are both less than $\widetilde{Q}/n$, the third case is that the marginal cost and marginal value have two intersections and one intersection is less than $\widetilde{Q}/n$ while the other is greater than $\widetilde{Q}/n$. All three cases are shown in Figure 6. When the marginal cost is always greater than the marginal value, each member will acquire no private information. However, when the marginal cost and marginal value have two intersections, the larger intersection point can be one candidate for the information acquisition in equilibrium.

Another comment about Figure 6 is that the relationship between the marginal cost and marginal value is changing in the committee size as long as $C'(0) < v(\widetilde{Q})$. Notice that when the committee size goes larger, the maximizer of the marginal value for each committee member is monotonically decreasing. Therefore, for some cost

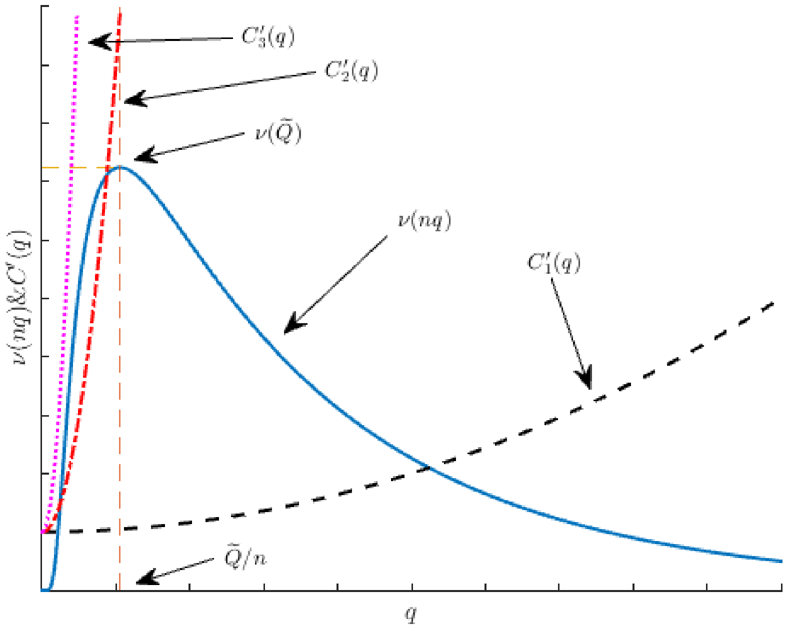


Figure 6: Information Acquisition When $\Lambda \neq 1$

functions, when the committee size is small enough, marginal cost is always greater than the marginal value, such as $C_3(q)$ in Figure 6. As committee size increases, the marginal cost and marginal value may have two intersections and the intersections will be less than $\widetilde{Q}/n$. When the committee size continues to increase, one of the intersection will be larger than $\widetilde{Q}/n$. Therefore, when the committee size is large enough, the marginal cost and marginal value will have two intersections as long as $C'(0) < \nu(\widetilde{Q})$, and when the committee size becomes even larger one of the two intersections will be larger than $\widetilde{Q}/n$.

Weibull et al. (2007) and Lindbeck and Weibull (2016) have shown us that when the value of information is non-concave, the larger intersection *might not* be an equilibrium; the information acquisition in Equation (33) is local maximum, the payoff from the information acquisition *might* be less than the payoff without information acquisition. However we can show that when the committee size is large enough, members will acquire some positive private information and equilibrium information will be determined by the first-order condition.

Figure 7 explains why committee size determines if the first-order condition gives a local maximum or global maximum. Note that given $\hat{\boldsymbol{\varphi}}$, the payoff of committee

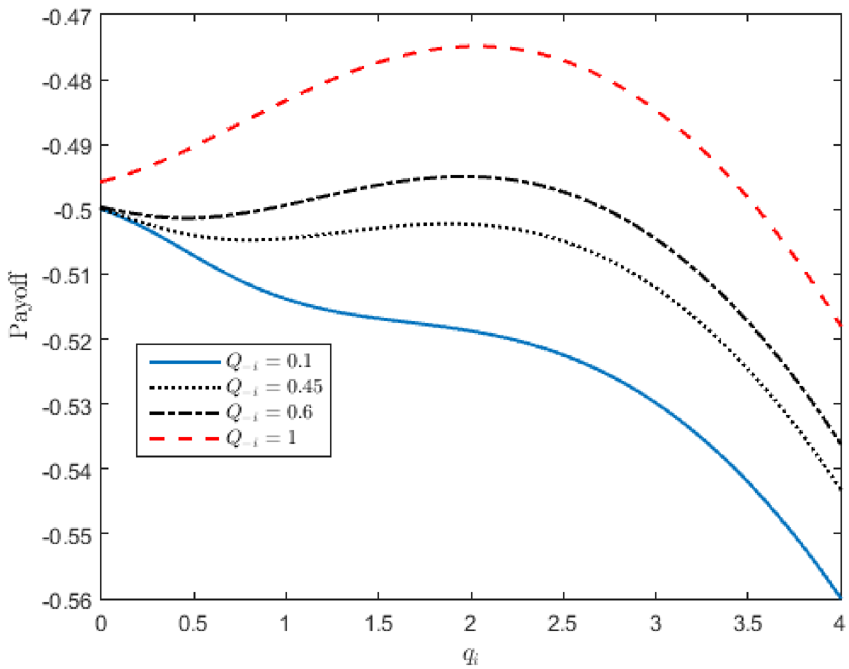


Figure 7: $v_i(\boldsymbol{\varphi}^*, q_i, \mathbf{q}_{-i})$ with Different Q_{-i}

member i is

$$v_i(\hat{\boldsymbol{\varphi}}, q_i, \mathbf{q}_{-i}) = -\alpha\gamma\Phi[(s^* - 1)\sqrt{Q_{-i} + q_i}] - \beta(1-\gamma)\left\{1 - \Phi[s^*\sqrt{Q_{-i} + q_i}]\right\} - C(q_i)$$

where $Q_{-i} = \sum_{j \neq i} q_j$. Figure 7 shows that when Q_{-i} is *not* large enough, the first-order condition is local maximum and the global maximum is $q_i = 0$. When Q_{-i} is large enough, the first-order condition gives the global maximum. Suppose $Q^*_{-i} = Q^* - q^*_i$ is monotonically increasing in the committee size n. Then when n is large enough, Q^*_{-i} is large enough to guarantee that the first-order condition implies the global maximum. As will be shown in Propositions 9 and 10, when members determine their equilibrium information acquisition according to the first-order condition, Q^*_{-i} is monotonically increasing in the committee size. This leads us to the following proposition.

PROPOSITION 8. *Suppose* $\Lambda \neq 1$. *Then*

(i) if $C'(0) < \nu(\widetilde{Q})$, *there exists an* $\tilde{n}$ *such that when* $n \geq \tilde{n}$ *there are infinite many equilibria in the game* Γ_ψ. *In each equilibrium the committee member* $i \in \mathcal{I}$ *reports*

$$r_i = \varphi_i(s_i, q_i) = a_i s_i + b_i \tag{30}$$

where

$$a_i = \lambda \cdot q_i \text{ with } \lambda \in \mathbb{R}_{++} \tag{31}$$

and b_i *satisfies*

$$\sum_{i=1}^{n} b_i = nR - \frac{\lambda}{2} Q - \lambda \ln \Lambda \tag{32}$$

and acquires the private information q^* *which is uniquely determined by*

$$q^* = \sup\{q : v(Q) = C'(q)\} \tag{33}$$

where $Q = nq$*;*

(ii) if $C'(0) \geq v(\widetilde{Q})$*, each committee member acquires no private information and the society chooses* $d = 0 (d = 1)$ *if and only if* $\Lambda > 1 (\Lambda < 1)$*.*

From Equations (30) to (32) we know that, with *a priori* imbalance committee members will still adjust their report according to the decision rule threshold R and the optimal decision threshold s^*. This adjustment makes sure that the decision rule cannot affect the final decision. Therefore, the information acquisition choice in equilibrium is independent of the decision rule. It is defined by the first-order condition that the marginal value equals to the marginal cost, the optimal information acquisition is shown in Figure 6.

Proposition 8 shows that when $C'(0) \geq v(\widetilde{Q})$, the marginal cost is always larger than the marginal value and therefore members will not acquire any private information. When $C'(0) < v(\widetilde{Q})$ there will be two intersections between the marginal cost and marginal value and the larger intersection will be global maximum as the committee size increases.

One more comment about Proposition 8 is that under some parameters and cost functions, there *might* be two optimal information acquisition choices: one is zero information acquisition and the other is determined by Equation (33). This is when the local maximum payoff in Figure 7 equals to the payoff without information acquisition. Actually, Weibull et al. (2007) and Lindbeck and Weibull (2016) have shown us that when the value of information is non-concave, there are two optimal information choice for some type since in their model the types are continuous. In Figure 7 there exists one Q_{-i} such that there are two global maximum. If for some parameters and the cost function Q^*_{-i} results in two global maximum of the committee member i, then in equilibrium there are two information acquisition, resulting in the same payoff.

Now let us have a look at each member's information acquisition in equilibrium. From Figure 6 we know that when $C'(0) < \nu(\widetilde{Q})$ and the committee size is large enough, the larger intersection of the marginal cost and marginal value will be larger than $\widetilde{Q}/n$, where each member's marginal value is monotonically decreasing. This is similar to the *a priori* imbalance model. Therefore, we have the following proposition.

PROPOSITION 9. *Suppose $\Lambda \neq 1$ and $C'(0) < \nu(\widetilde{Q})$. Then there exists one $\bar{n} \geq \tilde{n}$ such that when $n \geq \bar{n}$,*

$$\frac{dq^*(n)}{dn} < 0$$

and

$$\lim_{n \to +\infty} q^*(n) = 0.$$

The intuition of Proposition 9 is similar to Proposition 4: when $n \geq \tilde{n}$, the marginal value of information intersects with the marginal cost in the interval $[\widetilde{Q}/n, +\infty)$, in which the value of information is strictly concave and $\lim_{Q \to +\infty} \nu(Q) = 0$. In Section 4 we have shown that the decreasing marginal value of information will decrease each member's information acquisition in larger committees.

However in the *a priori* imbalance model, the marginal value of information may be increasing and both intersections will be in the interval $(0, \widetilde{Q}/n)$, in which the marginal value of information is convex and is monotonically decreasing in each member's information acquisition and committee size. Therefore when both intersections are less than $\widetilde{Q}/n$ and if the larger intersection is a solution in equilibrium, each member's information acquisition in equilibrium would be increasing in the committee size.

Since we have shown that the relationship between the marginal cost and marginal value is changing in the committee size, the monotonicity of each member's information acquisition in equilibrium may depend on the committee size for some cost functions satisfying $C'(0) < \nu(\widetilde{Q})$. Take $C_3(q)$ in Figure 6 as an example. When the committee size is very small, each member will not acquire any private information since the marginal cost is larger than the marginal value. When the committee size increases, the marginal cost and marginal value will have two intersections less than $\widetilde{Q}/n$ and if the larger intersection point is the equilibrium information acquisition, each member's information acquisition in equilibrium will be monotonically increasing in the committee size. When the committee size continues to increase, the larger intersection of the marginal cost and marginal value will be greater than $\widetilde{Q}/n$ and in this case each member's information acquisition in equilibrium is monotonically decreasing in the committee size.

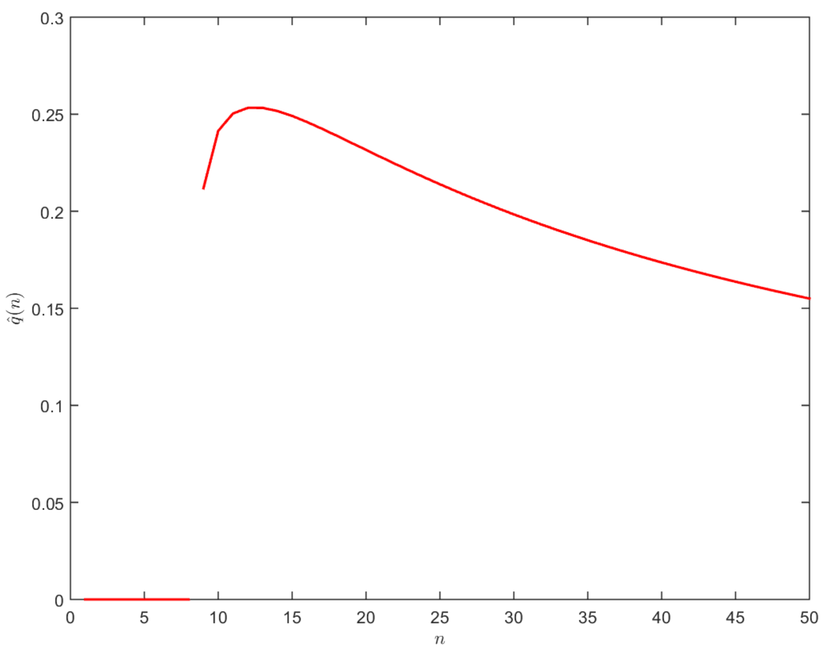


Figure 8: Information Acquisition in Equilibrium When $\Lambda \neq 1$

Figure 8 shows us one case in which the property of equilibrium information acquisition depends on the committee size. It shows the numerical solution when $\alpha = 5, \beta = 6$ and $\gamma = 0.1$, and the cost function is $C(q) = q^2/10$. We can see that $\Lambda = 10.8 > 1$ and individuals in the society, without information acquisition, are biased towards $d = 0$. Furthermore calculation shows that $\widetilde{Q} = 3.1623$ and $v(\widetilde{Q}) = 0.0507$. When the committee size is smaller than 8, the marginal cost is greater than the marginal value or the larger intersection is not a global maximum but a local maximum, each member will acquire zero private information. When the committee size is 9, each committee member starts to acquire private information, and since both intersections are less than $\widetilde{Q}/n$, the equilibrium information acquisition is monotonically increasing in the committee size. When the committee size continues to increase to be larger than 13, the larger intersection is larger than $\widetilde{Q}/n$ and therefore the equilibrium information acquisition is monotonically decreasing in the committee size. In this numerical simulation, $\bar{n}$ should be 13 while $\tilde{n}$ should be 9. In the *a priori* balance model, the equilibrium information acquisition should be monotonically increasing for all $n \in (\tilde{n}, \bar{n})$.

Different from the *a priori* balance model, Proposition 8 and Proposition 9 shows that rational ignorance exists in three cases: the first-one is $C'(0) \geq \nu(\widetilde{Q})$, the second one is $C'(0) < \nu(\widetilde{Q})$ and $n < \tilde{n}$, and the third one is $C'(0) < \nu(\widetilde{Q})$ and the committee size is large enough. Rational ignorance in the first two cases results from the non-concavity of the value of information, and the rational ignorance in the third one is due to the decreasing marginal value of information.

Furthermore, when $C'(0) < \nu(\widetilde{Q})$, the conclusions in Propositions 5 and 7 are still held in the model with asymmetric cost.

PROPOSITION 10. *Suppose* $\Lambda \neq 1$. *Then*

(i) if $C'(0) < \nu(\widetilde{Q})$ *and* $n \geq \tilde{n}$,

$$\frac{dQ^*(n)}{dn} \geq 0$$

with equality if and only if the information acquisition cost is linear;

(ii) if $C'(0) = 0$,

$$\lim_{n \to +\infty} Q^*(n) = +\infty$$

(iii) if $C'(0) = c \in (0, \nu(\widetilde{Q}))$,

$$\lim_{n \to +\infty} Q^*(n) = \nu^{-1}(c) > \widetilde{Q}$$

(iv) if $C'(0) = c \geq \nu(\widetilde{Q})$,

$$Q^*(n) = 0 \text{ for all } n.$$

The proof of the first point of Proposition 9 needs to distinguish between two cases. When the equilibrium information acquisition is less than $\widetilde{Q}/n$, the marginal value is monotonically increasing and therefore each member will acquire more private information in equilibrium; then it is intuitive that larger committees have more aggregate information. When the equilibrium information acquisition is more than $\widetilde{Q}/n$, then the intuition of Proposition 9 is the same as the intuition of Proposition 5. Therefore, Proposition 9 shows that to some extent the Condorcet Jury Theorem is satisfied.

Different from Proposition 5, when $C'(0) \geq \nu(\widetilde{Q})$, the committee, no matter what the size it is, is uninformative about the underlying state in the *a priori* imbalance model. They can only make the final decision according to the prior belief. On the

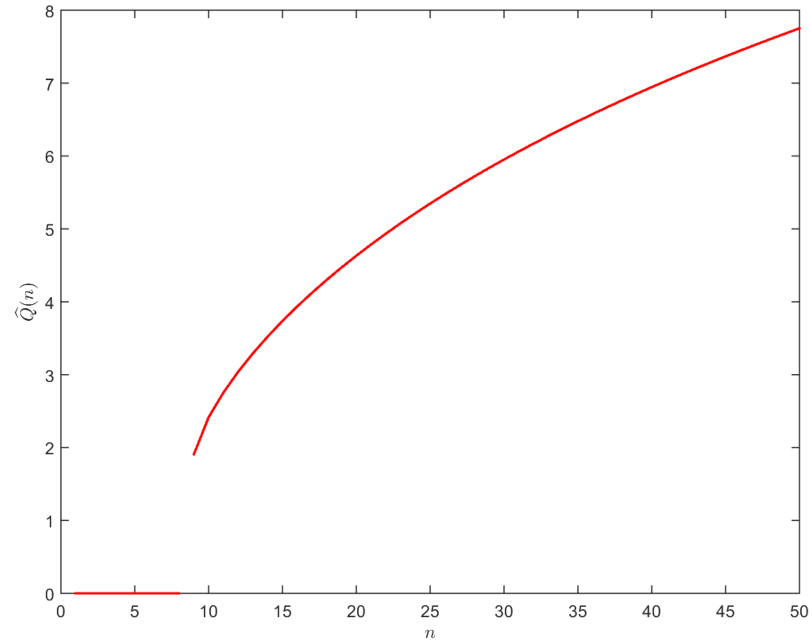


Figure 9: Aggregate Information in Equilibrium When $\Lambda \neq 1$

other hand, when $C'(0) < v(\widetilde{Q})$, the committee is still uninformative when the committee is *not* large enough. However, when $n \geq \tilde{n}$, the aggregate information gathered by committees will jump at $n = \tilde{n}$ and after the jump the aggregate information is non-decreasing in the size, and when the cost function is nonlinear it is strictly monotonically increasing in the size. Therefore, the aggregate information *might* be discontinuous in the size. Figure 9 shows the aggregate information when $\alpha = 5$, $\beta = 6$ and $\gamma = 0.1$ and the cost function is $C(q) = q^2/10$. When $n \leq 8$, it shows that the committee will *not* acquire any information, and when $n = 9$, aggregate information jumps from 0 to 1.9065. When $n \leq 9$ the aggregate information is monotonically increasing in the committee size.

This property of the aggregate information has some implications of the social welfare. From Proposition 10 and Figure 9 we know that when $C'(0) < v(\widetilde{Q})$, the best choice for the society is *not* to form the committee when $N < \tilde{n}$ since the final decision is made according to the prior belief. Furthermore Proposition 10 shows that when $C'(0) \geq v(\widetilde{Q})$, the committees are uninformative about the underlying state; therefore when there is participation cost the society should not form a committee to make the decision.

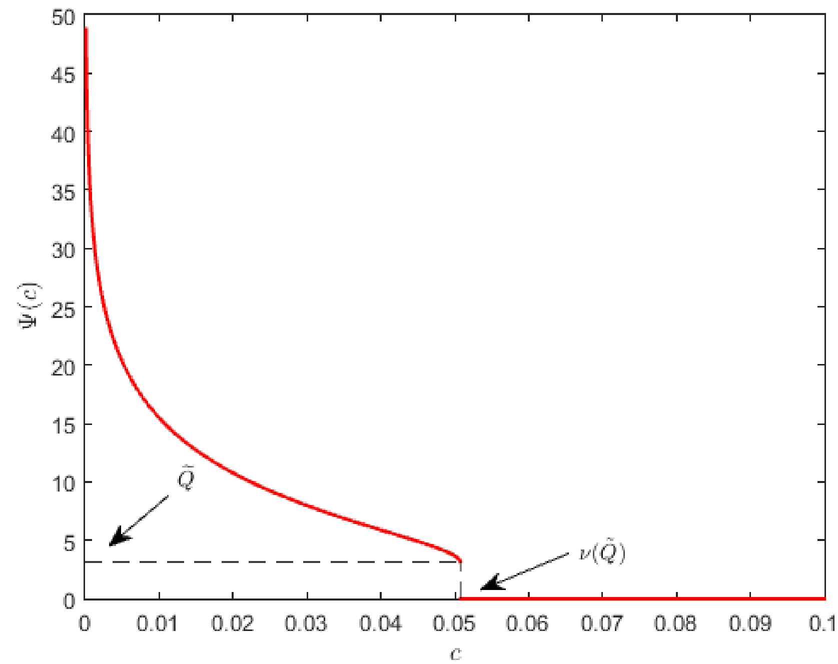


Figure 10: Asymptotic Aggregate Information When $\Lambda \neq 1$

Proposition 10 also shows that the asymptotic property of the equilibrium aggregate information in Proposition 7 still holds. This is based on the condition that $C'(0) < \nu(\widetilde{Q})$. If $C'(0) > \nu(\widetilde{Q})$ the committees, no matter what the size it is, will be uninformative about the underlying state. On the contrary, when $C'(0) = 0$ the committees will tend to make the appropriate decision with probability one when its size goes to infinity. Furthermore when $0 < C'(0) < \nu(\widetilde{Q})$ the aggregate information gathered by committees is bounded from above by $\nu^{-1}(c)$.

Since $\lim_{c \to 0} \nu^{-1}(c) = +\infty$, we can investigate the property of the function $\Psi(c)$, which is defined by Equation (29). From Proposition 10 we know that it is discontinuous. Figure 10 shows that when $c < \nu(\widetilde{Q})$ aggregate information is decreasing from $+\infty$ to $\widetilde{Q}$, and then after $c \geq \nu(\widetilde{Q})$ committees will be uninformative about the underlying state. From the figure committees can only either choose to acquire more than $\widetilde{Q}$ or be uninformative.

REMARK 2. *Suppose $\Lambda \neq 1$. Note that the social marginal value of information equals to the marginal benefit. This implies that the first-best information acquisition q^* in the* a priori *imbalance model is:*

(i) if $C'(0)/N > \nu(\widetilde{Q})$, $\hat{q} = 0$;

(ii) if $C'(0)/N \leq V(\widetilde{Q})$, there exists an n^* such that for all committee size $n \geq n^*$, the first-best information acquisition $\hat{q}$ is uniquely determined by $\hat{q} = \sup\{q : v(Q) = C'(q)/N\}$.

7 Extensions

In this section I want to extend the model from three aspects: first of all, I will show that the limit of probability of the appropriate decision goes to 1 if and only if the marginal cost at zero information acquisition is zero when the committee members can report 0 or 1; then I will show that the conclusions in the above sections holds when committee members have heterogeneous cost functions; finally I will check if the conclusions are still applicable for more general continuous distributions.

7.1 Strategic Voting

In this subsection I suppose that each committee member can only report 0 or 1, and that the final decision follows the τ-rule:

$$\psi_s(\mathbf{r}) = \begin{cases} 0, & \text{if } \frac{r_1+r_2+\cdots+r_n}{n} < \tau \\ 1, & \text{if } \frac{r_1+r_2+\cdots+r_n}{n} \geq \tau \end{cases} \tag{34}$$

where $\tau \in (0,1)$ and $n\tau$ is an integer.[9]

Following Austen-Smith and Banks (1996) and Feddersen and Pesendorfer (1996, 1997), we see that there is strategic voting. According to Li et al. (2001) and Duggan and Martinelli (2001) after information acquisition there exists a cutoff equilibrium such that each member reports following

$$r_i = \begin{cases} 0, & \text{if } s_i < t_i^* \\ 1, & \text{if } s_i \geq t_i^* \end{cases}$$

We want to solve for the symmetric equilibrium such that all committee members acquire the same private information q^* and follow the same report strategy characterized by the threshold t^*.

[9]Here we ignore the unanimity rule. Duggan and Martinelli (2001) have shown that the solution of unanimity rule is different from other rules. Furthermore, the assumption $n\tau$ being an integer is for notation convenience, and if it is not, replace $n\tau$ by $\lceil n\tau \rceil$ and all other calculations are the same.

Now suppose all members except for member i follow the strategy (q^*, t^*), then the payoff of player i is

$$\begin{aligned}\Pr[piv|\omega = &0]u(d=1,\omega=0)\Phi[-t_i\sqrt{q_i}]\Pr[\omega=0] \\ &+\Pr[piv|\omega=1]u(d=0,\omega=1)\Phi[(t_i-1)\sqrt{q_i}]\Pr[\omega=1]-C(q_i)\end{aligned} \tag{35}$$

plus a constant that is independent of player i's strategy. In the above expression,

$$\Pr[piv|\omega] = \binom{n-1}{n\tau-1}\Phi[-(t^*-\omega)\sqrt{q^*}]^{n\tau-1}\Phi[(t^*-\omega)\sqrt{q^*}]^{n-n\tau}$$

is committee member i's conditional probability of being pivotal given the underlying state $\omega \in \{0,1\}$.

Taking derivative of Equation (35) w.r.t. t_i, we can see that a necessary condition for an optimal threshold for member i is given by

$$J_\tau(n,t_i) = 0 \tag{36}$$

where

$$J_\tau(n,t_i) \triangleq \left[\frac{\Phi[-(t^*-1)\sqrt{q^*}}{\Phi[-t^*\sqrt{q^*}]}]\right]^{n\tau-1}\left[\frac{\Phi[(t^*-1)\sqrt{q^*}]}{\Phi[t^*\sqrt{q^*}]}\right]^{n-n\tau}\frac{\phi[(t_i-1)\sqrt{q_i}]}{\phi[t_i\sqrt{q_i}]}-\Lambda \tag{37}$$

Furthermore, Equation (35) shows that the marginal value of information acquisition of member i is given by

$$\begin{aligned}V_n(q_i) \triangleq &-\Pr[piv|\omega=0]u(d=1,\omega=0)\phi[t_i\sqrt{q_i}]\Pr[\omega=0]\frac{t_i}{2\sqrt{q_i}} \\ &+\Pr[piv|\omega=1]u(d=0,\omega=1)\phi[(t_i-1)\sqrt{q_i}]\Pr[\omega=1]\frac{t_i-1}{2\sqrt{q_i}}\end{aligned}$$

Then we can show:

LEMMA 2. $\lim_{n\to+\infty}\Pr[piv|\omega=0] = \lim_{n\to+\infty}\Pr[piv|\omega=1] = 0.$

That is to say, the probability of being pivotal tends to zero as the committee size goes to infinity. This implies that $\lim_{n\to+\infty}V_n(q_i) = 0$. Therefore, if $C'(0) > 0$, and if the committee size is large enough, the marginal value of information acquisition is strictly smaller than the marginal cost. Therefore,

PROPOSITION 11. *Suppose $C'(0) > 0$ and the reporting space is $\{0,1\}$. There exists an $\tilde{n}$ such that for all $n \geq \tilde{n}$, $q^*(n) = 0$, and*

$$\lim_{n\to+\infty}\Pr[d=1|\omega=0] > 0 \quad \textit{and} \quad \lim_{n\to+\infty}\Pr[d=0|\omega=1] > 0$$

Since the marginal value of information is strictly smaller than the marginal cost when the committee size is large enough, committee members have no incentive to acquire any private information. In this case, each committee member would report 0 when $\Lambda > 1$. Therefore the limit of probability of the appropriate decision is strictly less than 1 as long as the marginal cost at zero information acquisition is positive.

Now suppose $C'(0) = 0$, then for any marginal value of information acquisition, there is a signal precision q_i such that $V_n(q_i) = C'(q_i)$. About the symmetric equilibrium we have the following conclusions:

LEMMA 3. *Suppose $C'(0) = 0$ and the reporting space is $\{0, 1\}$. There exists an $\bar{n}$ such that for all $n \geq \bar{n}$ each committee member reports following*

$$\forall i, r_i = \begin{cases} 0, & \text{if } s_i < t^* \\ 1, & \text{if } s_i \geq t^* \end{cases}$$

where the threshold t^* is implicitly defined as:

$$t^* = \frac{1}{2} + \frac{\ln \Lambda + (n\tau - 1) \ln \left[\frac{\Phi[-t^*\sqrt{q^*}]}{\Phi[-(t^*-1)\sqrt{q^*}]} \right] + (n - n\tau) \ln \left[\frac{\Phi[t^*\sqrt{q^*}]}{\Phi[(t^*-1)\sqrt{q^*}]} \right]}{q^*} \tag{38}$$

and the information acquisition choice q^* is implicitly defined as:

$$V_n(q^*) = \binom{n-1}{n\tau - 1} \frac{\beta(1-\gamma)\Phi[-t^*\sqrt{q^*}]^{n\tau-1}\Phi[t^*\sqrt{q^*}]^{n-n\tau}\phi[t^*\sqrt{q^*}]}{2\sqrt{q^*}} = C'(q^*) \tag{39}$$

The threshold in Equation (38) is a solution of Equation (36): the existence of the threshold has been proved by Duggan and Martinelli (2001). Equation (39) solves the equilibrium information acquisition by equating the marginal value to the marginal cost. Note that as the committee size goes to infinity, the marginal value tends to zero; this implies that the equilibrium information acquisition tends to zero as the committee size goes to infinity. This is consistent with the rational ignorance, which is shown in Proposition 12. The existence of the solution in Equation (39) is guaranteed by the conclusion that

$$\lim_{q \to 0} V_n(q) = +\infty$$

and

$$\lim_{q \to +\infty} V_n(q) = 0$$

One more condition for the positive information acquisition is that the payoff with information acquisition is greater than $\max\{-\alpha\gamma, -\beta(1-\gamma)\}$, this is guaranteed by the conclusion in Proposition 12. Since when the committee size is large enough, the probability of the appropriate decision is very close to 1 and the cost paid by each member tends to zero, it is always beneficial for each member to acquire some private information when the committee size is large enough.

Before the next proposition, we have the following lemma.

LEMMA 4.
$$\widetilde{H} = \lim_{n \to +\infty} \sup \frac{\phi[\sqrt{q^*}(t^*-1)]}{\phi[\sqrt{q^*}t^*]} \frac{\Phi[-(t^*-1)\sqrt{q^*}]}{\Phi[-t^*\sqrt{q^*}]}$$
is finite.

The next proposition shows that the rational ignorance applies and the Condorcet Jury Theorem is applicable as long as the marginal cost at zero information acquisition is zero.

PROPOSITION 12. *Suppose $C'(0) = 0$ and the reporting space is $\{0,1\}$. Then*

(i) $\lim_{n \to +\infty} q^*(n) = 0$;

(ii) $\lim_{n \to +\infty} \Pr[d = 1|\omega = 0] = \lim_{n \to +\infty} \Pr[d = 0|\omega = 1] = 0$.

The first conclusion in the proposition shows that the rational ignorance theorem still applies in the strategic voting model when the signal is continuous. This is very intuitive since we have shown that the marginal value of information tends to zero as the committee size goes to infinity.

The second conclusion shows that the Condorcet Jury Theorem is applicable as long as the marginal cost at zero information acquisition is zero. The proof of the second part follows the idea of Duggan and Martinelli (2001). In the proof we show that Equations (36) and (37) and Lemma 4 imply

$$\lim_{n \to +\infty} \left(\frac{\Phi[-(t^*-1)\sqrt{q^*}]}{\Phi[-t^*\sqrt{q^*}]} \right)^{\tau} \left(\frac{\Phi[(t^*-1)\sqrt{q^*}]}{\Phi[t^*\sqrt{q^*}]} \right)^{1-\tau} = 1$$

and the above equation implies

$$\lim_{n \to +\infty} \Phi[-t^*\sqrt{q^*}] < \tau < \lim_{n \to +\infty} \Phi[-(t^*-1)\sqrt{q^*}]$$

Therefore, when $\omega = 0$, the probability of each committee member reporting 1 is less than τ and by the strong law of large numbers, the ratio of members reporting 1 is less than τ, indicating that limit of probability of choosing $d = 1$ is 0. Similarly, the above inequalities also show that the probability of each committee member reporting 1 is larger than τ when the underlying state is $\omega = 1$. By the strong law of large numbers the ratio of members reporting 1 is larger than τ; this indicates that the limit of probability of choosing $d = 1$ is 1.

We can compare the conclusions in Proposition 12 with the conclusions in Martinelli (2006). Martinelli (2006), in a strategic voting model with information acquisition, shows that the limit of probability of the appropriate decision goes to 1 if and only if both the marginal cost and the second order derivative of the cost function at zero information acquisition are zero. In our conclusions that the marginal cost at zero information acquisition is zero is a sufficient and necessary condition for the Condorcet Jury Theorem to hold. The only difference between our model and the model in Martinelli (2006) is that Martinelli (2006)'s signal space is binary while our signal space is $\mathbb{R}$. Therefore informative voting in Martinelli (2006) implies that each member votes $d = 1$ $(d = 0)$ if and only if the signal is 1 (0), while in our model each committee member needs to adjust the threshold according to the equilibrium information acquisition and the committee size.

7.2 Heterogeneous Information Acquisition

In this subsection I want to extend the results into the balance model with heterogeneous information acquisition cost functions. Formally, I suppose that each individual's information acquisition cost function is from the set $\{C(q,k)\}$, which is indexed by the parameter $\kappa \in K \triangleq [\underline{k}, \bar{k}]$. k represents the information acquisition skill. The cost function satisfies the condition that $\partial^2 C(q,k)/\partial q \partial k \geq 0$, which implies that increasing k will increase the marginal cost of information acquisition, and $\partial C(q,k)/\partial k \geq 0$. The distribution of the parameter is $H : K \to [0,1]$.[10] Denote $\mathbf{k}_n$ as the skill profile with n committee members, and $\mathbf{k}_{n+1} = (\mathbf{k}_n, k_{n+1})$ is the skill profile with $n+1$ members, and the first n members' skill is $\mathbf{k}_n$.

According to our method the cost function will not affect the reporting strategy in equilibrium in Proposition 3, the reporting equilibrium is the same as in Proposition 3. Therefore the marginal benefit will not change and the information acquisition is determined by the first-order condition that the marginal benefit equals to the marginal cost. Formally,

[10] $\bar{k}$ can be either finite or infinite and the distribution H can be either discrete or continuous.

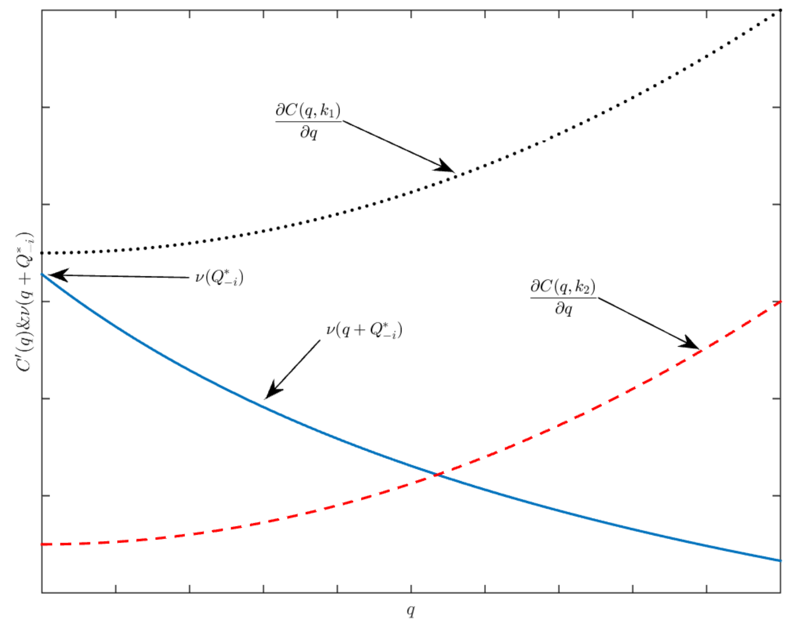


Figure 11: Information Acquisition with Heterogeneous Information Costs

LEMMA 5. *Suppose $\Lambda = 1$ and the reporting space is $\mathbb{R}$. Then there is a threshold $k^*(n, \mathbf{k}_n)$ such that in any linear equilibrium there is when $k_i > k^*(n, \mathbf{k}_n)$, $q_i^*(n, \mathbf{k}_n) = 0$; and when $k_i \leq k^*(n, \mathbf{k}_n)$, $q_i^*(n, \mathbf{k}_n) = q_i$, where q_i is uniquely determined by*

$$\nu(q_i + Q^*_{-i}(n, \mathbf{k}_n)) = \frac{\partial C(q_i, k_i)}{\partial q_i}$$

where $Q^*_{-i}(n, \mathbf{k}_n) = \sum_{j \neq i} q_j^*(n, \mathbf{k}_n)$.

The intuition of the lemma is that individual i will acquire positive information if and only if $\nu(Q^*_{-i}(n, \mathbf{k}_n)) > \frac{\partial C(q_i,k_i)}{\partial q_i}|_{q_i=0}$. Furthermore, we know that $\frac{\partial C(q_i,k_i)}{\partial q_i}|_{q_i=0}$ is non-decreasing in k_i. Then when k_i is enough, the marginal cost at zero information acquisition may be too high for the member to acquire any information. This process is shown in Figure 11: if committee member i's skill parameter is k_1, $C_q(0, k_1) > \nu(Q^*_{-i})$ and committee member i has no incentive to acquire any private information; if committee member i's skill parameter is k_2, $\frac{\partial C(q_1,k_1)}{\partial q_1}|_{q_1=0} < \nu(Q^*_{-1})$ and there is one unique intersection between the marginal cost function and marginal benefit function, committee member i will acquire a positve private information.

Furthermore, it is intuitive that when $n = 1$, $\kappa^*(1, \mathbf{k}_1) = \bar{k}$, which implies that no matter what the cost function is, the member in the one-member committee will acquire positive information. Suppose one more member is selected. If $C_q(0,k_2) > v(q_1^*)$, then we have $k^*(2,\mathbf{k}_2) < \bar{k}$. More generally we have shown in the appendix that

$$k^*(n+1,\mathbf{k}_{n+1}) \leq k^*(n,\mathbf{k}_n) \text{ for all } n \in \mathbb{N}$$

Therefore with more members the threshold of the skill will be non-increasing.

Denote[11]

$$\mathcal{K} \triangleq \left\{ k : \frac{\partial C(q,k)}{\partial q}|_{q=0} = \frac{\partial C(q,\underline{k})}{\partial q}|_{q=0} \right\} \tag{40}$$

as the set of skill parameter whose marginal cost at zero information acquisition equals to that of $\underline{k}$. We can see that it is nonempty since $\underline{k} \in \mathcal{K}$. For some cost functions the set is a singleton while for some others $\mathcal{K} = K$. Take $C(q,k) = kq^2$ where $k \in \mathbb{R}_{++}$ for example, we have $\mathcal{K} = K$.

Given the information acquisition in *Lemma* 5, we can see that the *ex-ante* aggregate information acquisition with committee size n is

$$\mathbb{E}[Q^*(n)] = \int_{K^n} Q^*(n,\mathbf{k}_n) dH(\mathbf{k}_n) \tag{41}$$

Then

PROPOSITION 13. *Suppose* $\Lambda = 1$ *and individuals in the society have heterogeneous information cost functions. Then*

(i) $q_i^*(n+1,\mathbf{k}_{n+1}) \leq q_i^*(n,\mathbf{k}_n)$ *for all* $i \in \mathcal{I}$ *and* $n \in \mathbb{N}$, *and if* $\Pr[k \in \mathcal{K}] > 0$, *there is*

$$\lim_{n\to+\infty} q_i^*(n,\mathbf{k}_n) = 0 \text{ for all } i \in \mathcal{I}$$

(ii) $Q^*(n,\mathbf{k}_n) \leq Q^*(n+1,\mathbf{k}_{n+1})$ *for all* $i \in \mathcal{I}$ *and* $n \in \mathbb{N}$, *and therefore*

$$\frac{d\mathbb{E}[Q^*(n)]}{dn} > 0$$

(iii) if $\Pr[k \in \mathcal{K}] > 0$ *and* $\frac{\partial C(q,\underline{k})}{\partial q}|_{q=0} = 0$,*we have*

$$\lim_{n\to+\infty} \mathbb{E}[Q^*(n)] = +\infty$$

[11]The conclusion in this section can be extended into the model where the cost function is parameterized by multiple parameters. For example, if the cost function is $C(q,k^1,k^2,\cdots,k^m)$ satisfying $\partial^2 C(q,k^1,\cdots,k^m)/\partial q \partial k^j \geq 0$ for $k^j \in [\underline{k}^j,\bar{k}^j]$, then the set can be defined as

$$\mathcal{K} \triangleq \left\{ (k^1,\cdots,k^m) : \frac{\partial C(q,k^1,\cdots,k^m)}{\partial q}|_{q=0} = \frac{\partial C(q,\underline{k}^1,\cdots,\underline{k}^m)}{\partial q}|_{q=0} \right\}$$

And then the conclusions in Proposition 13 can be extended in a similar way.

(iv) if $\Pr[k \in \mathcal{K}] > 0$ *and* $\frac{\partial C(q,\underline{k})}{\partial q}|_{q=0} = c > 0$*,we have*

$$\lim_{n \to +\infty} \mathbb{E}[Q^*(n)] = \nu^{-1}(c)$$

The first part of Proposition 13 shows that the Down's rational ignorance still holds when the cost functions are heterogeneous among individuals in the society. Furthermore, as we show in the appendix, when $k_{n+1} \geq k^*(n, \mathbf{k}_n)$, the participation of committee member $n+1$ would not change the others' information acquisition choice. While if $k_{n+1} < k^*(n, \mathbf{k}_n)$, the participation of committee member $n+1$ will move k^* downwards, and therefore decrease each member's information acquisition. Furthermore, combined with the conclusions in part (iii) and (iv) it is intuitive that when $\Pr[k \in \mathcal{K}] > 0$, the limit of each member's information acquisition is 0 as the size of the committee tends to be infinite.

The second part of the proposition shows that the ex-ante aggregate information acquisition is larger in larger committee. Intuitively, when $k_{n+1} \geq k^*(n, \mathbf{k}_n)$, the participation of member $n+1$ will not change the aggregate information acquisition. However, if $k_{n+1} < k^*(n, \mathbf{k}_n)$, then either members with less marginal cost or more members acquire positive private information, the aggregate information is determined to increase. Since one more member into the committee will either not change or increase the aggregate information, it is intuitive that the ex-ante aggregate information acquisition is monotonically increasing in the committee size.

Part (iii) and (iv) of the proposition studies the asymptotic properties of the information acquisition when there are heterogeneous cost functions. It shows that the asymptotic property is determined by the distribution of the skill parameters and the marginal cost at zero information acquisition with lowest skill parameter. If $\frac{\partial C(q,\underline{k})}{\partial q}|_{q=0} = 0$ and $\Pr[k \in \mathcal{K}] > 0$, then there is

$$\lim_{n \to +\infty} Q^*(n, \mathbf{k}_n) = +\infty$$

since there are infinite members with skill parameters whose marginal cost at zero information acquisition is zero in the profile $\mathbf{k}_n$: if the limit is finite, than all members whose skill parameter is in the set $\mathcal{K}$ will acquire positive private information. Since every skill profile will lead to the infinite information acquisition when the size goes to infinity, it is intuitive that the limit of the ex-ante aggregate information acquisition is infinite. Therefore, if the cost function set is $C(q,k) = kq^2$ with $k \in \mathbb{R}_{++}$ being distributed with distribution H, then the probability of making the appropriate decision goes to one when the committee size goes to infinity since $\Pr[k \in \mathcal{K}] = 1$.

Similarly, when $\frac{\partial C(q,k)}{\partial q}|_{q=0} = c > 0$ and $\Pr[k \in \mathcal{K}] > 0$, there is

$$\lim_{n\to+\infty} Q^*(n, \mathbf{k}_n) = \nu^{-1}(c)$$

since otherwise the members whose skill parameter is in the set $\mathcal{K}$ acquire positive private information. From Equation (41) we know that the ex-ante aggregate information will approach to $\nu^{-1}(c)$ when the committee size goes to infinity. One example is

$$C(q,k) = kqe^q$$

and $k \in \{\underline{k}, \bar{k}\}$ with $\Pr[k = \underline{k}] = 1 - \Pr[k = \bar{k}] = \pi > 0$, then Part (iv) of the proposition shows that

$$\lim_{n\to+\infty} Q^*(n, \mathbf{k}_n) = \lim_{n\to+\infty} \mathbf{E}[Q^*(n)] = \nu^{-1}(\underline{k})$$

REMARK 3. *Suppose $\Lambda \neq 1$. Then it is possible that in equilibrium all members acquire no private information. One example is that all cost functions are similar to $C_3(q)$ shown in Figure 6. However, if $\frac{\partial C(q,k)}{\partial q}|_{q=\widetilde{Q}} < \nu(\widetilde{Q})$ for all $k \in K$, there is an $\bar{n}$ and a threshold $k^*(n, \mathbf{k}_n)$ such that when $n \geq \bar{n}$, then if $k_i > k^*(n, \mathbf{k}_n)$, the equilibrium information acquisition is $q_i^*(n, \mathbf{k}_n) = 0$, and if $k_i \leq k^*(n, \mathbf{k}_n)$, $q_i^*(n, \mathbf{k}_n) = q$ where q is determined by*

$$\nu(q_i + Q_{-i}^*) = \frac{\partial C(q_i, k_i)}{\partial q_i}$$

Furthermore, all properties shown in Proposition 13 are also satisfied in the unbalanced model with heterogeneous cost functions.

7.3 General Continuous Distributions

In the above analysis we assume the normal conditional distributions of signals. In this subsection I want to extend the analysis into other continuous distributions. Formally I assume the conditional PDFs $f(s_i|\omega = 0; q_i)$ and $f(s_i|\omega = 1; q_i)$ are both continuous in s_i and q_i; they have the same support $(\underline{S}, \overline{S})$ where $\underline{S}, \overline{S} \in [-\infty, +\infty]$. I assume that the conditional distributions ahve mean ω and precision q_i. Furthermore, the conditional PDFs satisfy the monotone likelihood ratio property:

ASSUMPTION 1. *The likelihood ratio, $f(s|\omega = 1; q)/f(s|\omega = 0; q)$, is weakly increasing on s for all $s \in (\underline{S}, \overline{S})$.*

With these assumptions I want to see if the conclusions about the Condorcet Jury Theorem are still applicable when the reporting space is $\mathbb{R}$ and the society follows the average decision rule shown in Equation (15). Then I want to check if the conclusions are still applicable in the strategic voting model and the society follows the τ-rule.

First of all, suppose the society follows the average decision rule and the reporting space is $\mathbb{R}$. Then note that we are trying to solve the symmetric linear equilibrium, in which each agent's report function is linear in its own signal and all members acquire the same private information. The distribution of the average reports is determined by the average of all signals. However, according to Lindeberg-Lévy Central Limit Theorem,[12]

$$\sqrt{n}\left(\frac{1}{n}\sum_{i=1}^{n} s_i - \omega\right)|\omega \xrightarrow{d} \mathcal{N}\left(0, \frac{1}{q}\right)$$

This implies that in equilibrium

$$\lim_{n\to+\infty} \mathbb{E}[u(\psi(\mathbf{r}^*), \omega)] = -\alpha\gamma\Phi[(s^*-1)\sqrt{Q}] - \beta(1-\gamma)\Phi[-s^*\sqrt{Q}]$$

Therefore, when the committee size is large enough, the marginal value of information acquisition is very close to $\nu(Q)$. According to this we have the following conclusions:

PROPOSITION 14. *Suppose the continuous conditional PDFs are $f(s_i|\omega = 0; q_i)$ and $f(s_i|\omega = 1; q_i)$ and the reporting space is $\mathbb{R}$.*

(i) $\lim_{n\to+\infty} q^*(n) = 0$;

(ii) If $C'(0) = 0$, then

$$\lim_{n\to+\infty} Q^*(n) = +\infty \quad \textit{and} \quad \lim_{n\to+\infty} \Pr[d=0|\omega=0] = \lim_{n\to+\infty} \Pr[d=1|\omega=1] = 1;$$

(iii) If $C'(0) = c > 0$, then

$$\lim_{n\to+\infty} Q^*(n) = \nu^{-1}(c) \textit{ and } \lim_{n\to+\infty} \Pr[d=0|\omega=0] < 1 \textit{ and } \lim_{n\to+\infty} \Pr[d=1|\omega=1] < 1.$$

Since in the limit the marginal value of information acquisition is close to $\nu(Q)$, each member tends to acquire no private information when the committee size goes to infinity. The second and third points in Proposition 14 follow the same intuition as in Proposition 7.

[12]From the proof of Proposition 3 we know that the average signals being normally distributed is sufficient for the existence of linear equilibrium. Since the average of signals converges to a normal distribution, when the committee size is very large, there exists symmetric linear equilibria.

Now I want to test the Condorcet Jury Theorem if each committee member can only report 0 and 1, and the society makes the final decision following τ-rule in Equation (34). Note that the payoff of committee member i is

$$\Pr[piv|\omega=0]u(d=1,\omega=0)[1-F(t_i|\omega=0;q_i)]\Pr[\omega=0] \\ +\Pr[piv|\omega=1]u(d=0,\omega=1)F(t_i|\omega=1;q_i)\Pr[\omega=1]-C(q_i)$$

plus a constant independent of i's strategy. The conditional probability of being pivotal is

$$\Pr[piv|\omega]=\binom{n-1}{n\tau-1}[1-F(t^*|\omega,q)]^{n\tau-1}F(t^*|\omega,q)^{n-n\tau}$$

The equation for the threshold now is

$$J_{\tau,f}(n,t^*)=0$$

where

$$J_{\tau,f}(n)\triangleq\left[\frac{1-F(t^*|\omega=1;q)}{1-F(t^*|\omega=0;q)}\right]^{n\tau-1}\left[\frac{F(t^*|\omega=1;q)}{F(t^*|\omega=0;q)}\right]^{n-n\tau}\frac{f(t^*|\omega=1;q)}{f(t^*|\omega=0;q)}-\Lambda$$

Duggan and Martinelli (2001) have proved the existence of the threshold for given precision q and ASSUMPTION 1.

The marginal value of information acquisition is:

$$V_n(q)=-\Pr[piv|\omega=0]u(d=1,\omega=0)\Pr[\omega=0]\frac{\partial F(t^*|\omega=0;q)}{\partial q} \\ +\Pr[piv|\omega=1]u(d=0,\omega=1)\Pr[\omega=1]\frac{\partial F(t^*|\omega=1;q)}{\partial q}$$

Note that with continuous PDFs, there is

$$\lim_{n\to+\infty}\Pr[piv|\omega=1]=\lim_{n\to+\infty}\Pr[piv|\omega=0]=0$$

This implies that

$$\lim_{n\to+\infty}V_n(q)=0$$

Therefore, when $C'(0)>0$, and the committee size is large enough, there is no symmetric equilibrium with positive information acquisition. All members make the decision according to the common prior and therefore the limit of probability of the appropriate decision will be strictly less than 1. Furthermore, the limit of marginal value of information being zero implies that committee members tend to acquire zero information even when $C'(0)=0$. Therefore, we have the following conclusions:

PROPOSITION 15. *Suppose the continuous conditional PDFs are $f(s_i|\omega = 0; q_i)$ and $f(s_i|\omega = 1; q_i)$ and the reporting space is $\{0, 1\}$.*

(i) $\lim_{n\to+\infty} q^*(n) = 0$*;*

(ii) If $C'(0) = c > 0$, then

$$\lim_{n\to+\infty} \Pr[d = 0|\omega = 0] < 1 \text{ and } \lim_{n\to+\infty} \Pr[d = 1|\omega = 1] < 1;$$

(iii) If $C'(0) = 0$, then

$$\lim_{n\to+\infty} \Pr[d = 0|\omega = 0] = \lim_{n\to+\infty} \Pr[d = 1|\omega = 1] = 1.$$

The above proposition shows the conclusions about the Condorcet Jury Theorem with information acquisition from the strategic voting model are still applicable for more general continuous distributions satisfying the monotone likelihood ratio property. When $C'(0) = c > 0$ and the committee is large enough, members have no incentive to acquire any private information and therefore the limit of the probability of the appropriate decision is strictly less than 1. When $C'(0) = 0$, we can prove the third point of the above proposition following the same idea as in Proposition 12. From the equation for the threshold we can prove that in equilibrium

$$\lim_{n\to+\infty} 1 - F(t^*|\omega = 0; q^*) < \tau < \lim_{n\to+\infty} 1 - F(t^*|\omega = 1; q^*)$$

Therefore the strong law of large numbers implies that the limit probability of the right decision tend to be 1 when $C'(0) = 0$. Furthermore since the probability of the right decision tend to be 1, and each member's information acquisition tends to zero, each member can enjoy the low probability of the wrong decision by paying a little cost when the committee size is large enough and therefore the in equilibrium the information acquisition is determined by the first order condition that the marginal value equals to marginal cost when the committee size is large enough.

8 Conclusion

In a model where there is no interest conflict among individuals but the information is costly, we show that committee members have less incentive to acquire private information in larger committees when the committee size is large enough and each committee member tends to acquire zero information when the committee size goes to infinity. However the aggregate information gathered by committees is increasing in committee size. Therefore the probability of making the appropriate decision is larger in larger committees and the Condorcet Jury Theorem is partly satisfied. Furthermore, whether the probability of making the appropriate decision tends to one when the size goes to infinity depends on the information acquisition cost function. We show that aggregate information gathered by committees tends to infinity when the size goes to infinity if and only if the marginal cost at zero information acquisition is zero. If the marginal cost at zero information acquisition is positive, the aggregate information is bounded from above and therefore there is some probability of making the wrong decision even when there are infinite members in the committee; in this case the Condorcet Jury Theorem is *not* satisfied.

The present model is very parsimonious; it only considers the normal distribution and all individuals have the same preference. In real life individuals have interest conflicts and their information acquisition may not be normally distributed; furthermore their reports may not be fully revealed but partly revealed. Hence it would be interesting to investigate the Condorcet Jury Theorem following five avenues.

Firstly, we assume a certain type of continuous distribution. It is natural to extend the analysis into the model with flexible information acquisition, shown in Yang (2015). I think since the model with flexible information acquisition model excludes the effects of the signal distribution and the information acquisition is in the form of entropy, we can have more general conclusions from this kind of model.

Secondly, in our analysis we solve the equilibria in which the private information is fully revealed and the reporting space is binary. However Li et al. (2001) has shown us that with no interest conflicts there are multiple partition equilibria, with partition categories changing from 2 to infinity. We have proved the results with 2 partition categories and infinite partition categories, but there should be infinitely many other partition categories with certain decision rules. Generally more partition categories imply there is fewer information loss during information transmission. we have proved the two extreme cases, and therefore we can conjecture that whether the Condorcet Jury Theorem is satisfied depends on the properties of information acquisition cost function in other partition equilibria, but we have had no proof yet.

Thirdly, it is very common that there are interest conflicts among individuals in reality. Li et al. (2001) has shown us that members have incentives to manipulate information when there are preference conflicts, and therefore partition equilibrium is the only monotone equilibria. Depending on the disagreement zone, the data partition may be different. It would be very interesting to introduce the preference conflicts and study the Condorcet Jury Theorem in the model with heterogeneous preferences. However from Section 7 we see that in the equilibrium with two partitions, the marginal value of information acquisition tends to zero as the committee size goes to infinity and there is an equation for the threshold; therefore we can conjecture that the limit of the probability of the right decision tends to one if and only if the marginal cost at zero information acquisition is 0 and the limit is strictly less than 1 if and only if the marginal cost at zero information acquisition is positive.

Fourthly, in this model we assume signals are independent; in reality signals may be correlated. Ladha (1992, 1993) have studied the effects of correlation between signal on the Condorcet Jury Theorem in a naive voting model. Few research has extended the analysis into the strategic voting models and the reporting models in our paper. It would be interesting to extend the analysis in this aspect.

Finally there is no participation cost in our model while in reality there are. When there are participation costs, members may choose to abstain. McMurray (2013) has shown that the quality of the private signals will affect whether or not to participate or abstain in a voting game. It would very interesting to extend the analysis into our model. We can predict that the participation cost and the choice of abstain will affect the information acquisition in equilibrium, and therefore the Condorcet Jury theorem with information acquisition.

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Appendix

Proof of Proposition 3

Given the information acquisition profile $\mathbf{q}$, suppose committee member j with $j \neq i$ reports

$$r_j = \varphi_j(s_j, q_j) = a_j s_j + b_j$$

then given the conditional distribution of s_j, there is

$$r_j|\omega \sim \mathcal{N}\left(a_j\omega + b_j, \frac{a_j^2}{q_j}\right)$$

Therefore,

$$\frac{\sum_{j\neq i} r_j}{n} \sim \mathcal{N}\left(\frac{\sum_{j\neq i} a_j}{n}\omega + \frac{\sum_{j\neq i} b_j}{n}, \frac{1}{n^2}\sum_{j\neq i}\frac{a_j^2}{q_j}\right)$$

Furthermore, given signal s_i and the report strategy profile $\boldsymbol{\varphi}(\mathbf{s}, \mathbf{q}) = \mathbf{r}$, the benefit for committee member i is

$$\begin{aligned}
\mathbb{E}[u(d,\omega)|s_i;\mathbf{q}] &= \int_{\mathbf{S}_{-i}\times\Omega} u(\psi(\boldsymbol{\varphi}(\mathbf{s},\mathbf{q})),\omega)dF(\mathbf{s}_{-i},\omega|s_i;q_i,\mathbf{q}_{-i}) \\
&= \int_{\mathbf{S}_{-i}} \left\{\gamma\frac{f(\mathbf{s}|\omega=1;\mathbf{q})}{f(s_i;q_i)}u(\psi(\mathbf{r}_{-i},r_i),1) + (1-\gamma)\frac{f(\mathbf{s}|\omega=0;\mathbf{q})}{f(s_i;q_i)}u(\psi(\mathbf{r}_{-i},r_i),0)\right\} d\mathbf{s}_{-i} \\
&= -\alpha\gamma\frac{f(s_i|\omega=1;q_i)}{f(s_i,q_i)}\Phi\left[\frac{n}{\sqrt{\sum_{j\neq i} a_j^2/q_j}}\left(R - \frac{r_i}{n} - \frac{\sum_{j\neq i} a_j}{n} - \frac{\sum_{j\neq i} b_j}{n}\right)\right] \\
&\qquad - \beta(1-\gamma)\frac{f(s_i|\omega=0;q_i)}{f(s_i,q_i)}\left\{1 - \Phi\left[\frac{n}{\sqrt{\sum_{j\neq i} a_j^2/q_j}}\left(R - \frac{r_i}{n} - \frac{\sum_{j\neq i} b_j}{n}\right)\right]\right\}
\end{aligned}$$

where $f(s_i;q_i) = \Pr[\omega=1]f(s_i|\omega=1;q_i) + \Pr[\omega=0]f(s_i|\omega=0;q_i)$ is the unconditional probability density function of signal s_i.

The optimal report for member i given other members' reports is

$$\hat{\varphi}_i \in \arg\max_{r_i \in \mathbb{R}} \left\{ \mathbb{E}[u(\psi(\boldsymbol{\varphi}_{-i}(\mathbf{s}_{-i}, \mathbf{q}_{-i}), r_i), \omega) | s_i, \mathbf{q}] \right\}$$

The FOC implies

$$\alpha\gamma \frac{f(s_i, |\omega = 1; q_i)}{f(s_i; q_i)} \phi \left[\frac{n}{\sqrt{\sum_{j \neq i} a_j^2 / q_j}} \left(R - \frac{r_i}{n} - \frac{\sum_{j \neq i} a_j}{n} - \frac{\sum_{j \neq i} b_j}{n} \right) \right] =$$

$$\beta(1-\gamma) \frac{f(s_i | \omega = 0; q_i)}{f(s_i; q_i)} \phi \left[\frac{n}{\sqrt{\sum_{j \neq i} a_j^2 / q_j}} \left(R - \frac{r_i}{n} - \frac{\sum_{j \neq i} b_j}{n} \right) \right]$$

Or equivalently,

$$r_i = \frac{q_i \sum_{j \neq i} a_j^2 / q_j}{\sum_{j \neq i} a_j} s_j + nR - \sum_{j \neq i} b_j - \frac{1}{2} \sum_{j \neq i} a_j - \frac{\sum_{j \neq i} a_j^2 / q_j}{\sum_{j \neq i} a_j} \left(\ln \Lambda + \frac{q_i}{2} \right)$$

Therefore given the report of all members except for i's are linear in their signals, the best response of member i's report should also be linear in his signal s_i. Suppose $r_i = \varphi_i(s_i, q_i) = a_i s_i + b_i$, then there is

$$a_i = \frac{q_i \sum_{j \neq i} a_j^2 / q_j}{\sum_{j \neq i} a_j} \tag{42}$$

$$b_i = nR - \sum_{j \neq i} b_j - \frac{1}{2} \sum_{j \neq i} a_j - \frac{\sum_{j \neq i} a_j^2 / q_j}{\sum_{j \neq i} a_j} \left(\ln \Lambda + \frac{q_i}{2} \right) \tag{43}$$

From our assumption we know that the above equation system is right for all $i \in \{1, 2, \cdots, n\}$. From Equation (42) we have

$$a_i = q_i \frac{\sum_{j=1}^n a_j^2 / q_j}{\sum_{j=1}^n a_j} \tag{44}$$

From Equation (44) we know the solution for a_i with $i = 1, 2, \cdots, n$ is

$$a_i = \lambda \cdot q_i \text{ with } \lambda \in \mathbb{R}_{++} \tag{45}$$

Plugging Equation (45) into Equation (43) we have

$$\sum_{i=1}^n b_i = nR - \frac{\lambda}{2} Q - \lambda \ln \Lambda \tag{46}$$

The optimal information acquisition for committee member i is

$$q_i \in \arg\max_{q_i} \left\{ \int_{S_i} \mathbb{E}[u(\psi(\hat{\boldsymbol{\varphi}}_{-i}(\mathbf{s}_{-i}, \hat{\mathbf{q}}_{-i}), \hat{\varphi}_i(s_i, q_i)), \omega) | s_i; q_i, \hat{\mathbf{q}}_{-i}] - C(q_i) \right\}$$

The goal function can now be simplified as

$$-\alpha\gamma\Phi\left[\sqrt{Q}\left(\frac{\ln\Lambda}{Q} - \frac{1}{2}\right)\right] - \beta(1-\gamma)\left\{1 - \Phi\left[\sqrt{Q}\left(\frac{\ln\Lambda}{Q} + \frac{1}{2}\right)\right]\right\} - C(q_i)$$

The first order condition implies

$$\begin{aligned} &\alpha\gamma\phi\left[\sqrt{Q}\left(\frac{\ln\Lambda}{Q} - \frac{1}{2}\right)\right]\left[\frac{1}{2\sqrt{Q}}\left(-\frac{\ln\Lambda}{Q} + \frac{1}{2}\right) + Q^{-3/2}\ln\Lambda\right] \\ &\quad + \beta(1-\gamma)\phi\left[\sqrt{Q}\left(\frac{\ln\Lambda}{Q} + \frac{1}{2}\right)\right]\left[\frac{1}{2\sqrt{Q}}\left(\frac{\ln\Lambda}{Q} + \frac{1}{2}\right) - Q^{-3/2}\ln\Lambda\right] = C'(q_i) \end{aligned}$$

Note that $s^* = 1/2 + \ln\Lambda/Q$ and $\alpha\gamma\phi[(s^* - 1)\sqrt{Q}] = \beta(1-\gamma)\phi\left[s^*\sqrt{Q}\right]$, the above equation can be simplified as

$$\nu(Q) \triangleq \frac{\beta(1-\gamma)\phi[s^*\sqrt{Q}]}{2\sqrt{Q}} = C'(q_i)$$

Furthermore, the monotonicity of $\nu(Q)$ guarantees the existence and uniqueness of the solution in the above equation.

Furthermore note that $\nu''(Q) < 0$, so there is

$$\nu''(Q) - C''(q) < 0$$

which suggests that the goal function is concave in $[0, +\infty)$. Therefore the unique solution of the first-order condition maximizes the goal function.

Proof of Corollary 2

First of all, the social marginal value of information acquisition is larger than the marginal value of information acquisition, committee members will *not* acquire sufficient information.

Secondly, we have

$$\frac{d(\hat{q}/q^*)}{dN} = \frac{q^* d\hat{q}/dN}{(q^*)^2} > 0$$

where the first equation applies the fact that $dq^*/dN = 0$ and the inequality follows the fact that

$$\frac{d\hat{q}}{dN} = \frac{\nu(Q)}{C''(\hat{q}) - Nn\frac{d\nu(Q)}{dQ}} > 0$$

since $\nu(Q)$ is monotonically decreasing in Q given $\Lambda = 1$.

Proof of Proposition 4

See the proof in Propositions 5 and 7.

Proof of Proposition 5

From Equation (18), we know that when $C(q) = cq$, the aggregate information gathered by the committee is

$$Q^* = \nu^{-1}(c)$$

and each individual acquires the private information $q^*(n) = \nu^{-1}(c)/n$, which is decreasing in n and goes to zero as the size n goes to infinity.

When the information acquisition cost function is non-linear, suppose on the contrary that aggregate information gathered by the committee is decreasing in the committee size, i.e., $dQ^*(n)/dn \leq 0$, then we have

$$\frac{d\nu(Q^*)}{dn} = \frac{d\nu(Q^*)}{dQ^*}\frac{dQ^*(n)}{dn} \geq 0$$

and therefore from Equation (18) we have

$$\frac{dq^*(n)}{dn} \geq 0$$

and therefore $Q^*(n) = nq^*(n)$ is monotonically increasing in the size n, a contradiction. Therefore we have

$$\frac{d\nu(Q^*)}{dn} < 0$$

and

$$\frac{dq^*(n)}{dn} < 0$$

this is consistent with the conclusion in Proposition 4.

Proof of Proposition 6

Since the decision is determined by the information gathered by the committee, we can denote each individual's ex-ante utility by

$$u(Q^*(n)) \triangleq \mathbb{E}u(d(Q^*(n)), \omega) = -\alpha\gamma\Phi\left[(s^* - 1)\sqrt{Q^*(n)}\right] - \beta(1-\gamma)\Phi\left[s^*\sqrt{Q^*(n)}\right] \tag{47}$$

Then, the ex-ante social welfare is

$$W(n,N) \equiv u(Q^*(n)) - \frac{C(q^*(n))}{N}$$

When the information acquisition cost function is linear, Proposition 5 has shown that the aggregate information gathered by the committee does not change in its size, we have $du(Q^*(n))/dn = 0$; furthermore, the aggregate cost borne by the committee is $nC(q^*(n)) = cQ^*$, constant in the size. Therefore, the social welfare is constant in the committee size.

When the information acquisition cost function is nonlinear, we have

$$\begin{aligned}
&W(n,N) - W(n-1,N) \\
&= \frac{(n-1)[C(q^*(n-1)) - C(q^*(n))] + N[u(Q^*(n)) - u(Q^*(n-1))] - C(q^*(n))}{N} \\
&> [u(Q^*(n)) - u(Q^*(n-1))] - \frac{C(q^*(n))}{N}
\end{aligned}$$

where the last inequality follows the fact that $q^*(n-1) > q^*(n)$ in equilibrium.

The last expression is positive as long as

$$N > N_0 \triangleq \frac{C(q^*(n))}{u(Q^*(n)) - u(Q^*(n-1))}$$

Proof of Proposition 7

When $C'(0) = 0$, and if

$$\lim_{n \to +\infty} Q^*(n) < +\infty$$

then there is $\widetilde{Q} > 0$ such that

$$\lim_{n \to +\infty} Q^*(n) = \widetilde{Q}$$

then we have

$$\lim_{n \to +\infty} v(Q^*(n)) = \frac{\beta(1-\gamma)\phi\left(s^*\sqrt{\widetilde{Q}}\right)}{2\sqrt{\widetilde{Q}}} > 0$$

so from Equation (18) there is a $\tilde{q} > 0$ such that

$$\lim_{n \to +\infty} q^*(n) = \tilde{q}$$

this implies

$$\lim_{n \to +\infty} Q^*(n) = \lim_{n \to +\infty} \tilde{q}(n) = +\infty$$

a contradiction.

Furthermore, we have

$$\lim_{n\to+\infty} \nu(Q^*(n)) = \lim_{Q^*(n)\to+\infty} \nu(Q) = 0$$

and so

$$\lim_{n\to+\infty} q^*(n) = 0$$

When $C(q) = cq$ with $c > 0$, we have shown that $Q^*(n) = \nu^{-1}(c)$ and $q^*(n) = \nu^{-1}(c)/n$ for all $n \in \mathbb{N}$.

When $C'(0) = c > 0$ and the cost function is nonlinear, suppose on the contrary

$$\lim_{n\to+\infty} Q^*(n) \neq \nu^{-1}(c)$$

then there is a $\widetilde{Q}$ satisfying either $\widetilde{Q} > \nu^{-1}(c)$ or $\widetilde{Q} < \nu^{-1}(c)$ such that

$$\lim_{n\to+\infty} Q^*(n) = \widetilde{Q}$$

If $\widetilde{Q} < \nu^{-1}(c)$, then from the fact that $\nu(Q^*)$ is a monotonically decreasing function in Q^*, we have $c < \nu(\widetilde{Q})$. This implies that there is a $\tilde{q} > 0$ such that

$$\lim_{n\to+\infty} q^*(n) = \tilde{q}$$

therefore, we have

$$\lim_{n\to+\infty} Q^*(n) = +\infty$$

and

$$\lim_{n\to+\infty} \nu(Q^*(n)) = 0$$

it is not attainable according to the assumption on information acquisition cost function. If $\widetilde{Q} > \nu^{-1}(c)$, this implies $c > \nu(\widetilde{Q})$, it is not attainable since $C'(q) \geq c$ according to our assumption.

Since

$$\lim_{n\to+\infty} Q^*(n) = \nu^{-1}(c)$$

we have

$$\lim_{n\to+\infty} \nu(Q^*(n)) = c$$

and from Equation (18), this implies

$$\lim_{n\to+\infty} q^*(n) = 0.$$

Proof of Proposition 8

First of all, if $C'(0) < \nu(\widetilde{Q})$, we note that the reporting behavior in equilibrium is similar to Proposition 3, we can follow the same method to prove it and therefore it is omitted here. What needs to prove now is that when the committee size is large enough, the information acquisition in Equation (33) maximize the payoff. From Figure 6 we know that except for the information acquisition from the first-order condition, one competing choice is $q = 0$, which results in the payoff $\max\{-\beta(1-\gamma), -\alpha\gamma\}$. Remember that $\nu(Q)$ is positive and therefore the utility $\mathbb{E}u(d,\omega)$ is monotonically increasing in q, and $\lim_{n\to+\infty} C(q) = 0$, there exists one $\tilde{n}$ such that when $n \geq \tilde{n}$, the payoff with information acquisition is greater than the payoff without information acquisition.

Secondly, if $C'(0)geq\nu(\widetilde{Q})$, the cost function and the marginal value of the information does not have any intersection and therefore, the decision is determined by Λ, the cost of the two types of errors.

Proof of Lemma 2

In symmetric equilibria, the probability of being pivotal is given by

$$\Pr(piv|\omega = 1) = \binom{n}{n\tau - 1} \Phi[-(t-1)\sqrt{q}]^{n\tau-1}\Phi[(t-1)\sqrt{q}]^{n-n\tau}$$

$$\Pr(piv|\omega = 0) = \binom{n}{n\tau - 1} \Phi[-t\sqrt{q}]^{n\tau-1}\Phi[t\sqrt{q}]^{n-n\tau}$$

Note that for all $x \in [0,1]$, there is:

(i) if $\tau \in (0, \frac{1}{2}]$, then

$$(1-x)^{n\tau-1}x^{n-n\tau} = [x(1-x)]^{n\tau-1}x^{(1-2\tau)n+1} \leq [x(1-x)]^{n\tau-1} \leq 2^{2-2n\tau}$$

where the last inequality follows the fact that $x(1-x)$ is bounded from by $\frac{1}{4}$.

(ii) if $\tau \in (\frac{1}{2}, 1]$, then

$$(1-x)^{n\tau-1}x^{n-n\tau} = [x(1-x)]^{n(1-\tau)}(1-x)^{(2\tau-1)n-1} \leq [x(1-x)]^{n(1-\tau)} \leq 2^{2n(\tau-1)}$$

where the first inequality applies when n is large enough.

Therefore, for either $\omega = 0$ or $\omega = 1$,

$$\Pr(piv|\omega) \leq \frac{(n-1)!}{\max\{2^{2n\tau-2}, 2^{2n(1-\tau)}\}(n\tau-1)!(n-n\tau)!} \to 0$$

Proof of Lemma 3

The existence of the threshold has been proved by Duggan and Martinelli (2001) and therefore is ignored here. Now we want to prove the existence of the positive optimal information acquisition when the committee size is large enough.

First of all, note that in the proof of Lemma 4 we will show that the condition for threshold in Equations (36) and (37) imply

$$-\infty < t^* \sqrt{q^*} < +\infty$$

and

$$-\infty < t^* \sqrt{q^*} < +\infty$$

This implies that for given n,

$$\lim_{q \to +\infty} V_n(q) = 0$$

and

$$\lim_{q \to 0} V_n(q) = +\infty$$

So we have proved the existence of the solution in Equation (39). We need to prove that the solution in Equation (39) maximizing each committee member's payoff. To see this, note that Proposition 12 implies that the payoff for each member goes to zero as the committee size goes to infinity since the probability of the appropriate decision goes to one and the cost of each committee member goes to zero, this is larger than $\max\{-\alpha\gamma, -\beta(1-\gamma)\}$, the expected payoff without any information acquisition. Therefore, when the committee size is large enough, it is always beneficial to acquire some private information and the optimal information acquisition is given by the first order condition shown in Equation (39).

Proof of Lemma 4

For notation inconvenience we assume $t^* = t$ and $q^* = q$. Equation (37) can be expressed as:

$$\begin{aligned} J_\tau(n, t_i) &= \left(\frac{\Phi[-(t-1)\sqrt{q}]}{\Phi[-t\sqrt{q}]} \right)^{n\tau - 1} \left(\frac{\Phi[(t-1)\sqrt{q}]}{t\Phi[\sqrt{q}]} \right)^{n - n\tau} \frac{\phi[(t_i - 1)\sqrt{q_i}]}{t_i \phi[\sqrt{q_i}]} - \Lambda \\ &= [L_\tau(n)]^n \cdot \frac{\phi[(t_i - 1)\sqrt{q_i}] / \Phi[-(t-1)\sqrt{q}]}{\phi[t_i \sqrt{q_i}] / \Phi[-t\sqrt{q}]} - \Lambda \end{aligned}$$

where

$$L_\tau(n) = \left(\frac{\Phi[-(t-1)\sqrt{q}]}{\Phi[-t\sqrt{q}]}\right)^{\tau} \left(\frac{\Phi[(t-1)\sqrt{q}]}{\Phi[t\sqrt{q}]}\right)^{1-\tau}$$

Now suppose the lemma is wrong. Then there must be a subsequence of information acquisition and cutoff such that either $t\sqrt{q} \to +\infty$ or $t\sqrt{q} \to -\infty$. Without loss of generality, we assume that this is true for the whole subsequence.

First of all, if $t\sqrt{q} \to -\infty$, then

$$\lim_{n\to+\infty} \frac{\Phi[-(t-1)\sqrt{q}]}{\Phi[-t\sqrt{q}]} = 1$$

and

$$\lim_{n\to+\infty} \frac{\Phi[(t-1)\sqrt{q}]}{\Phi[t\sqrt{q}]} = \lim_{n\to+\infty} \frac{\phi[(t-1)\sqrt{q}]}{\phi[t\sqrt{q}]} = \lim_{n\to+\infty} \exp\left[\frac{q}{2}(2t-1)\right]$$
$$= \lim_{n\to+\infty} \exp[\sqrt{q}(\sqrt{q}t)] = +\infty$$

where the last condition comes from the assumption that $\widetilde{H}$ is infinite. This is one impossible since $t\sqrt{q} < 0$ and $\sqrt{q} \geq 0$.

Now suppose $t\sqrt{q} \to +\infty$, then

$$\lim_{n\to+\infty} \frac{\Phi[(t-1)\sqrt{q}]}{\Phi[t\sqrt{q}]} = 1$$

and

$$\lim_{n\to+\infty} \frac{\Phi[-(t-1)\sqrt{q}]}{\Phi[-t\sqrt{q}]} = \lim_{n\to+\infty} \frac{\phi[(t-1)\sqrt{q}]}{\phi[t\sqrt{q}]} = +\infty$$

where the last equality comes from the assumption that $\widetilde{H}$ is infinite.

Take any $c \in (0,1)$, then there exists m such that for all $n > m$,

$$\frac{\Phi[(t-1)\sqrt{q}]}{\Phi[t\sqrt{q}]} \geq c \qquad \text{and} \qquad \frac{\Phi[-(t-1)\sqrt{q}]}{\Phi[-t\sqrt{q}]} \geq \left(\frac{1}{c}\right)^{2(1-\tau)/\tau}$$

Therefore,

$$\left(\frac{\Phi[-(t-1)\sqrt{q}]}{\Phi[-t\sqrt{q}]}\right)^{n\tau-1} \left(\frac{\Phi[(t-1)\sqrt{q}]}{\Phi[t\sqrt{q}]}\right)^{n-n\tau} \geq c^{3-3n/2}c^{n/2} = c^{-n(1-\tau)+(1-\tau)/\tau} \to +\infty$$

Therefore, $J_\tau(n,t) > 0$ for high enough n, this violates the condition for the threshold in Equation (36).

Proof of Proposition 12

The first conclusion follows the idea that when n tends to infinity, the probability of being pivotal is zero and therefore the marginal value of information acquisition goes to zero.

Now let's prove the Condorcet Jury Theorem. This proof follows Duggan and Martinelli (2001). First of all, we need to prove $L_\tau(n) \to 1$. If not, then we can extract a subsequence with limitsup greater than one or limitinf less than one. Without loss of generality we assume that this is true for the whole sequence. If the limit of the sequence is to make $L_\tau(n) > 1$ for high enough n, then

$$\lim_{n\to+\infty} J_\tau(n,t) = \widetilde{H} \lim_{n\to+\infty} L_\tau(n)^n - \Lambda = \infty$$

This contradicts with the condition for the threshold in Equation (36).

If the limit of the subsequence is to make $L_\tau(n) < 1$ for high enough n, then

$$\lim_{n\to+\infty} J_\tau(n,t) = \widetilde{H} \lim_{n\to+\infty} L_\tau(n)^n - \Lambda = -\Lambda < 0$$

This contradicts with the condition for the threshold in Equation (36).

Now we want to prove that $L_\tau(n) = 1$ implies $\Phi[-t\sqrt{q}] < \tau < \Phi[-(t-1)\sqrt{q}]$. Note that the function $x^\tau(1-x)^{1-\tau}$ is monotonically increasing in $[0,\tau]$ and monotonically deceasing in $[\tau,1]$. Then if $\tau < \Phi[-t\sqrt{q}] < \Phi[-(t-1)\sqrt{q}]$, there is

$$\Phi[-t\sqrt{q}]^\tau \Phi[t\sqrt{q}]^{1-\tau} > \Phi[-(t-1)\sqrt{q}]^{1-\tau} \Phi[(t-1)\sqrt{q}]^\tau$$

This implies $L_\tau(n) < 1$, which contradicts with $L_\tau(n) = 1$. Similarly, if $\Phi[-t\sqrt{q}] < \Phi[-(t-1)\sqrt{q}] < \tau$, then $L_\tau(n) > 1$, which contradicts with $L_\tau(n) = 1$.

Since $\Phi[-t\sqrt{q}] < \tau < \Phi[-(t-1)\sqrt{q}]$, by the continuity of the normal distribution functions, we can take $\delta > 0$ such that for all $s' \in [t-\delta, t+\delta]$, there is $\Phi[-s'\sqrt{q}] < \tau < \Phi[-((s'-1))\sqrt{q}]$.

Now suppose $\omega = 1$. Define the sequence $X_1, X_2, \cdots$ of i.i.d. random variables satisfying

$$\forall i \quad X_i = \begin{cases} 1, \text{ if } s_i \geq t+\delta \\ 0, \text{ if } s_i < t+\delta \end{cases}$$

Then by the strong law of large numbers,

$$\frac{1}{n}\sum_{i=1}^{n} X_i \xrightarrow{a.s.} \Phi[-(t+\delta-1)\sqrt{q}]$$

In particular,

$$\Pr\left[\Phi[-(t+\delta-1)\sqrt{q}] - \frac{1}{n}\sum_{i=1}^{n} X_i > \epsilon\right] \longrightarrow 0$$

for all $\epsilon > 0$.

Now define the sequence of random variables $Y_1, Y_2, \cdots$, as

$$Y_n = \frac{1}{n}\#\{i \leq n | s_i \geq t\}$$

Clearly, $Y_n \geq \frac{1}{n}\sum_{i=1}^{n} X_i$ and therefore

$$\Pr\left[\Phi[-(t+\delta-1)\sqrt{q}] - Y_n > \epsilon\right] \longrightarrow 0$$

for all $\epsilon > 0$. Since we have shown that $\Phi[-(t+\delta-1)\sqrt{q}] > \tau$, we can define $\epsilon = \Phi[-(t+\delta-1)\sqrt{q}] - \tau > 0$ and therefore

$$\Pr[Y_n < \tau] \longrightarrow 0$$

This implies

$$\lim_{n\to+\infty} \Pr[d = 0 | \omega = 1] = 0$$

Now suppose $\omega = 0$. Define the sequence $X_1, X_2, \cdots$, of i.i.d. random variables satisfying

$$X_i = \begin{cases} 1, \text{ if } s_i \geq t-\delta \\ 0, \text{ if } s_i < t-\delta \end{cases}$$

Then by the strong law of large numbers,

$$\frac{1}{n}\sum_{i=1}^{n} X_i \xrightarrow{a.s.} \Phi[-(t-\delta)\sqrt{q}]$$

In particular,

$$\Pr[\frac{1}{n}\sum_{i=1}^{n} X_i - \Phi[-(t-\delta)\sqrt{q}] \geq \epsilon] \longrightarrow 0$$

for all $\epsilon > 0$.

Now define the sequence of random variables $Y_1, Y_2, \cdots$ as

$$Y_n = \frac{1}{n}\#\{i \leq n | s_i \geq t\}$$

Clearly, $Y_n \leq \frac{1}{n}\sum_{i=1}^{n} X_i$, and therefore

$$\Pr[Y_n - \Phi[-(t-\delta)\sqrt{q}] \geq \epsilon] \longrightarrow 0$$

Since $\Phi[-(t-\delta)\sqrt{q}] < \tau$, we can define $\varepsilon = \tau - \Phi[-(t-\delta)\sqrt{q}]$ and therefore the above equation implies

$$\Pr[Y_n \geq \tau] \longrightarrow 0$$

which implies

$$\lim_{n\to+\infty} \Pr[d = 1|\omega = 0] = 0$$

Proof of Lemma 5

Suppose $C_q(0, k_i) > \nu(0 + Q^*_{-i})$, the first order condition has no solution and therefore the best choice is not to acquire any private information.

Furthermore, according our assumption we know that $q_i^*(n, \mathbf{k}_{-i}, k_i)$ is non-increasing in k_i since marginal cost of information acquisition is non-decreasing in the skill parameter k. Therefore Q^*_{-i} is larger for larger k_i and therefore $\nu(0 + Q^*_{-i})$ is smaller for larger k_i. Therefore there is one $k^*(n, \boldsymbol{k}_n)$ such that when $k_i \geq k^*(n, \boldsymbol{k}_n)$, $C_q(0, k_i) \geq \nu(0 + Q^*_{-i})$ and when $k_i \leq k^*(n, \boldsymbol{k}_n)$, $C_q(0, k_i) < \nu(0 + Q^*_{-i})$.

Furthermore, when $n = 1$, there is $k^*(n, \boldsymbol{k}_n) = \bar{k}$ since there must be positive information acquisition with one-member committee. Now given the threshold $k^*(n, \boldsymbol{k}_n)$ given the committee size n and the skill profile $\boldsymbol{k}_n$; suppose there is one more member and denote its skill k_{n+1}. If $k_{n+1} \geq k^*(n, \boldsymbol{k}_n)$, then $q^*_{n+1}(n+1, \boldsymbol{k}_{n+1}) = 0$ and $k^*(n+1, \boldsymbol{k}_{n+1}) = k^*(n, \boldsymbol{k}_n)$. If $k_{n+1} < k^*(n, \boldsymbol{k}_n)$, then if the threshold k^* does not change, then $q^*_{n+1}(n+1, \boldsymbol{k}_{n+1}) > 0$ and for all $i \neq n+1$, there is $Q^*_{-i}(n+1, \boldsymbol{k}_{n+1}) > Q^*_{-i}(n, \boldsymbol{k}_n)$ will be increasing, and this would move the marginal benefit downwards, this contradicts with the assumption that the threshold k^* does not change. If $k^*(n+1, \boldsymbol{k}_{n+1}) > k^*(n, \boldsymbol{k}_n)$, then all members will increase its information acquisition, and this would reduce the marginal benefit of information acquisition; this contradicts with the assumption that $k^*(n+1, \boldsymbol{k}_{n+1}) > k^*(n, \boldsymbol{k}_n)$.

Proof of Proposition 13

First of all, note that $k^*(n, \boldsymbol{k}_n) \geq k^*(n+1, \boldsymbol{k}_{n+1})$, this implies that the marginal benefit of each committee member is non-increasing in n; therefore, there is $q^*(n, \boldsymbol{k}_n) \geq q^*_{n+1}(n+1, \boldsymbol{k}_{n+1})$.

Secondly, note that for all $Q^*(n, \mathbf{k}_n)$, if there is one more committee member with information acquisition skill k_{n+1}; then if $C_q(0, k_{n+1}) \geq \nu(0, Q^*(n, \boldsymbol{k}_n))$, there is $q^*(n + 1, \boldsymbol{k}_{n+1}) = 0$ and for all $i \neq n+1$, $q^*(n, \boldsymbol{k}_n) = q^*(n+1, \boldsymbol{k}_{n+1})$; and therefore $Q^*(n, \boldsymbol{k}_n) = Q^*(n+1, \boldsymbol{k}_{n+1})$. Now suppose $k_{n+1} < k^*(n, \boldsymbol{k}_n)$, we know that for all $i \neq n+1$, there is $q_i^*(n, \boldsymbol{k}_n) \geq q_i^*(n+1, \boldsymbol{k}_{n+1})$ with some i being strict inequality. From the first order condition we know that this implies $Q^*(n, \boldsymbol{k}_n) < Q^*(n+1, \boldsymbol{k}_{n+1})$. Therefore we have $\mathbb{E}[Q^*(n)]$ is monotonically increasing in n.

Since $\mathbb{E}[Q^*(n)]$ is monotonically increasing, then the limit of $\mathbb{E}[Q^*(n)]$ is either $+\infty$ or some finite positive real number. If $\Pr[k \in \mathcal{K}] > 0$, then when n goes to infinity, each skill profile $\boldsymbol{k}_n$ has infinite members whose marginal cost at zero information acquisition is 0.

If when $C_q(0, \underline{k}) = 0$ and $Q^*(\infty, \mathbf{k}_\infty) < +\infty$, then for all $k_i \in \mathcal{K}$, there is

$$C_q(0, k_i) < \nu(q_i + Q^*_{-i})$$

and therefore $q_i^*(\infty, \mathbf{k}_\infty) > 0$. This implies

$$\lim_{n \to +\infty} Q^*(n, \mathbf{k}_n) \geq \lim_{n \to +\infty} n \cdot \min \{q_i(n, \mathbf{k}_n) : k_i \in \mathcal{K}\} = +\infty$$

This contradicts with the assumption that $Q^*(\infty, \mathbf{k}_\infty) < +\infty$. Therefore, we have

$$\lim_{n \to +\infty} q_i^*(n, \mathbf{k}_n) = 0$$

since

$$C_q(0, k_i) > \nu(0 + Q^*_{-i}(\infty, \mathbf{k}_\infty)) \text{ for all } k_i \notin \mathcal{K}$$

and

$$C_q(0, k_i) = \nu(0 + Q^*_{-i}(\infty, \mathbf{k}_\infty)) \text{ for all } k_i \in \mathcal{K}$$

. Furthermore, according to Lebesgue's monotone convergence theorem, there is

$$\lim_{n \to +\infty} \mathbb{E}[Q^*(n)] = \int_{K^n} \lim_{n \to +\infty} Q^*(n, \mathbf{k}_n) dH(\mathbf{k}_n) = +\infty$$

Similarly, if $C_q(0, \underline{k}) = c > 0$, and $Q^*(\infty, \mathbf{k}_\infty) \neq \nu^{-1}(c)$. Then if $Q^*(\infty, \mathbf{k}_\infty) > \nu^{-1}(c)$, then for all $i \in \mathcal{I}$, there is $C_q(0, k_i) > \nu(0 + Q^*_{-i}(\infty, \mathbf{k}_\infty))$, and therefore $q_i^*(\infty, \mathbf{k}_\infty) = 0$, which implies $Q^*(\infty, \mathbf{k}_\infty) = 0$, which contradicts our assumption. On the contrary, if $Q^*(\infty, \mathbf{k}_\infty) < \nu^{-1}(c)$, then for all $i \in \mathcal{K}$, there is $C_q(0, k_i) < \nu(0 + Q^*_{-i}(\infty, \mathbf{k}_\infty))$ and therefore $q_i^*(\infty, \mathbf{k}_\infty) > 0$; therefore $Q^*(\infty, \mathbf{k}_\infty) = \infty$, which contradicts with out assumption. Therefore, we have

$$\lim_{n \to +\infty} q_i^*(n, \mathbf{k}_n) = 0$$

since

$$C_q(0,k_i) > \nu(0 + Q^*_{-i}(\infty, \mathbf{k}_\infty)) \text{ for all } k_i \notin \mathcal{K}$$

and

$$C_q(0,k_i) = \nu(0 + Q^*_{-i}(\infty, \mathbf{k}_\infty)) \text{ for all } k_i \in \mathcal{K}$$

. Furthermore, according to Lebesgue's monotone convergence theorem, there is

$$\lim_{n\to+\infty} \mathbb{E}[Q^*(n)] = \int_{K^n} \lim_{n\to+\infty} Q^*(n, \mathbf{k}_n) dH(\mathbf{k}_n) = \nu^{-1}(c)$$

Proof of Remark 3

Denote

$$\widetilde{\mathcal{I}} = \{i : q_i^*(n, \mathbf{k}_n) > 0\}$$

as the set of members whose information acquisition in equilibrium is positive.

Then for all $i \in \widetilde{\mathcal{I}}$, there should be

$$\nu(q_i + Q^*_{-i}(n, \mathbf{k}_n)) = \frac{\partial C(q_i, k_i)}{\partial q_i} \quad (48)$$

$$Q^*(n, \mathbf{k}_n) > \widetilde{Q} \quad (49)$$

$$\begin{aligned} -\alpha\gamma\Phi[(s^*-1)\sqrt{Q^*_{-i}+q_i}] - \beta(1-\gamma)\left\{1 - \Phi[s^*\sqrt{Q^*_{-i}+q_i^*}]\right\} - C(q_i) \\ \geq -\alpha\gamma\Phi[(s^*-1)\sqrt{Q^*_{-i}}] - \beta(1-\gamma)\left\{1 - \Phi[s^*\sqrt{Q^*_{-i}}]\right\} \end{aligned} \quad (50)$$

where Equation (48) is the first-order condition for information acquisition, Inequality (49) states that the aggregate information acquisition should be more than $\widetilde{Q}$, and Inequality (50) states that given other members' information acquisition in equilibrium, member i's optimal information acquisition should be larger than 0. The last two inequalities are derived from the non-concavities of the marginal benefit of information acquisition. As we have shown in Figure 7 the inequality is satisfied when Q^*_{-i} is large enough.

Note that according to the assumption there are solutions satisfying Equation (48) and Inequality (49). Furthermore, note that $\nu(Q)$ is decreasing in Q when $Q > \widetilde{Q}$, this implies that $Q^*(n, \mathbf{k}_n) \leq Q^*(n+1, \mathbf{k}_{n+1})$. Moreover, when n is large enough, Inequality (50) is satisfied. Therefore we can follow the proof of Lemma 5 and Proposition 13 to show the other part of the remark.